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MATHEMATICAL INVESTIGATION OF CALCULATION ZONES IN THE LARGE INDUSTRIAL EXCAVATIONS AND EMBANKMENTS

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Abstract

The paper presents mathematical method of investigation of "calculation zones". Described analysis are connected with realisation of investments, it means large excavations or embankments on the industrial areas. The presented method provides sufficient approximation of considered surfaces, in consequence, it can be used for cubature calculation of irregular or regular layers of large excavations or embankments. Calculation were carry out in FORTRAN language.

1. Introduction

The mentioned layers of large embankments or excavations request mathematical description. Reference points from individual layers, create base for mathematical investigation. Assumed that one layer creates "basic zone". It can be divided into "additional zones". The range of a given "basic zone" can be arbitrary selected. Division into "additional zones" is independent (Piasek, 2002).
Discussed zones create so called "calculation zones". In order to determine precisely a given calculation zone, the measuring points are represented by spatial co-ordinates, their alpha numerical names, codes, and a number of a zone. Assumed that reference points:
of the "basic zone" have value of code equal to zero,
of the first "additional zone" have code equal to 1, for the second zone equal to 2 and so on.
Coded points and sides in generated triangle net give possibility to use linear or non-linear function for mathematical approximation of analysed zones.

2. Mathematical investigation layers of embankments and excavations

Mathematical investigation of "calculation zones" requires solving problems as follows:
- zoning of excavations or embankments layers,
- automatic generation of irregular triangles net,
- mathematical approximation of a given zone,
- calculation of heights in points with given flat co-ordinates (checking points),
- calculation of cubature of earth masses.

The above problems have been solved in proposed system TRIANG (Piasek, 2001), (Piasek, 2002/1), (Piasek 2002/2). At the beginning, system TRIANG declares of matrices as 10th elementary chain of characters.
Called subprogram INP reads the following input data:
- co-ordinates and names of reference points from considered layers,
- current code of points from current generating zone. It is code of surface of calculation zones as well.

Next program generates irregular net of element - triangles (description in (Piasek, 2002/1)). At the stage of automatic generation the codes have been ascribed to the sides of triangles. If there is code =1, for the sides located inside calculation zone , the curvature parameters are handed over from considered triangle to the adjacent one. In such case, parameters are calculated for the elevation function of a higher degree. On the edge of calculation zone the codes of sides in generated triangles were assumed equal to 2. In consequence, the curvature parameters of the sides are equal to zero.

Layers interpretation of linear and non-linear shape function.

The linear elevation function has the following form (Piasek, 2001):

$$z = L_1 Z_1 + L_2 Z_2 + L_3 Z_3 \qquad (1)$$

where: $Z_{(N)}$ - elevation at the n - th triangle apex,

$L_{(N)}$ - functions of shape in triangle.

On the base of formula (1) we calculate elevations at the three apexes of triangle:

$$z(x_1, y_1) = L_1(x_1, y_1)Z_1 + L_2(x_1, y_1)Z_2 + L_3(x_1, y_1)Z_3$$
$$z(x_2 y_2) = L_1(x_2, y_2)Z_1 + L_2(x_2, y_2)Z_2 + L_3(x_2, y_2)Z_3$$
$$z(x_3, y_3) = L_1(x_3, y_3)Z_1 + L_2(x_3, y_3)Z_2 + L_3(x_3, y_3)Z_3$$

Definitions of shape functions are as follow:

at apex 1: $L_2(x_1, y_1) = 0 \ ; \ L_3(x_1, y_1) = 0 \ \ and \ \ L_1(x_1, y_1) = 1$ (2)

at apex 2: $L_1(x_2, y_2) = 0 \ ; \ L_3(x_2, y_2) = 0 \ \ and \ \ L_2(x_2, y_2) = 1$ (3)

at apex 3: $L_1(x_3, y_3) = 0 \ ; \ L_2(x_3, y_3) = 0 \ \ and \ \ L_3(x_3, y_3) = 1$ (4)

In order to simplify the calculations we assume shape functions in the following form:

$$L_1 = (a_1 + b_1 x + c_1 y) \ / 2\Delta$$
$$L_2 = (a_2 + b_2 x + c_2 y) \ / 2\Delta$$
$$L_3 = (a_3 + b_3 x + c_3 y) \ / 2\Delta$$

where: 2Δ - doubled surface area of a given triangle.

The a,b,c - coefficients multiplied by 2Δ.

From (Piasek, 2001) it can be seen that the simplest function, which assures simple interpretation of its parameters and effectiveness of numerical calculation is:

$$z = L_1 Z_1 + L_2 Z_2 + L_3 Z_3 + P_1 L_1 (L_2 + L_3) + P_2 L_2 (L_1 + L_3) + P_3 L_3 (L_1 + L_2) \qquad (5)$$

where:

Z_1, Z_2, Z_3 - constants,

P_1, P_2, P_3 - parameters.

We require a smooth transition of surface curvatures at the midpoint of side being common to adjacent triangles. Thus we shall calculate derivative at the triangle side midpoints.

Thus, the assumed forms of derivatives at the given side midpoints are:

$$\frac{\partial z}{\partial x} \Bigg|_{\substack{for \\ side : i \\ midpoint}} = P_i \frac{b_i}{2\Delta} + \frac{Z_1 b_1 + Z_2 b_2 + Z_3 b_3}{2\Delta} \qquad (6)$$

$$\frac{\partial z}{\partial y} \Bigg|_{\substack{for \\ side : i \\ midpoint}} = P_i \frac{b_i}{2\Delta} + \frac{Z_1 c_1 + Z_2 c_2 + Z_3 c_3}{2\Delta} \qquad (7)$$

The directional derivative of the function of two variables is expressed by the following formulas:

$$Z' \Bigg|_{\substack{in \\ direction V}} = \frac{\partial z}{\partial x} V_x + \frac{\partial z}{\partial y} V_y$$

where: V_x - x co-ordinate of vector V,

V_y - y co-ordinate of vector V.

Having formula for directional derivatives of elevation function for two adjacent triangles and at midpoint of the common side, we can compare them to the equation:

$$\frac{\partial z^m}{\partial x} V_x + \frac{\partial z^m}{\partial y} V_y = \frac{\partial z^n}{\partial x} V_x + \frac{\partial z^n}{\partial y} V_y$$

where: m - number of a considered triangle,

n - number of adjacent triangle.

Thus:

$$P_j^n = P_i^m \frac{T_i^m}{T_j^n} + \frac{S_i^m - S_j^n}{T_j^n} \tag{8}$$

Then we must establish another equation and calculate the unknown parameters. We shall apply the condition of the minimum sum of squared parameters:

$$(P_i^m)^2 + (P_j^n)^2 = MINIMUM$$

In this way, by balancing the parameter distribution we prevent the incorrect function of shape. Substituting formula for P_j^n into the above formula yields:

$$(P_i^m)^2 + (P_i^m \frac{T_i^m}{T_j^n})^2 + 2(P_i^m \frac{T_i^m (S_i^m - S_j^n)}{(T_j^n)^2}) + (\frac{S_i^m - S_j^n}{T_j^n}) = MINIMUM$$

For generated triangles parameters of linear (formula (1)) or non–linear (formula (5)) function to be calculated.

Next, we introduce the co-ordinates of the checking points with their quantity, the generation radius R, mean distance between the closed points from the given calculation zone, matrices of points in generated triangles and their quantity. Program locates the number of the triangles with checking points and calculates heights of considered points. Checking points in triangles will be located.

Program generates triangles on slope surface, which connect regular or irregular upper layer of designed embankment with terrain surface. This program extend quantity of generated triangles.

Numerical model of slope surface based on the numerical model terrain. In the selected points from the numerically presented layer, we multiple declarated value of slopes and calculate co-ordinates of unit vectors in accordance with theory presented in (Piasek, 2002/2).

Next calculation depends on parameter value (noted as P) in considered layer. If:

P = 1 program calculates parameters of curvatures for non-linear function,

P = 0 program calculates parameters for linear function.

Next, program calculates the cubature contained between considered two layers and profile co-ordinates by the aid regression function SPLINE.

For numerical integration of the function of two variables, we assume Simpson's method. This method needs compatibility of interpolation function *y=g(x)* with the given function *y=f(x)* in *n+1* chosen points.

The integral of function *y=f(x)* has form:

$$\int_{x_0}^{x_2} y dx = \frac{1}{3} h(\overline{y_0} + 4y_1 + y_2)$$

Now, we calculate double integral:

$$\iint f(x,y)dF = \int_{x_0}^{x_2} \left[\int_{y_0}^{y_2} f(x,y)dy \right] dx$$

We know that "x" (or "y") is constant if integrated function is relative to variable "y" (or "x") within the division: < y_0, y_2 > (or < x_0, x_2 >).

Thus:

$$\int_{x_0}^{x_2} \left[\frac{1}{3} h(f(x,y_0) + 4f(x,y_1) + f(x,y_2)) \right] dx =$$

$$= \frac{1}{3} h \left\{ \frac{1}{3} k \left[f(x_0,y_0) + 4f(x_0,y_1) + f(x_0,y_2) \right] + \right.$$

$$+4\left[\frac{1}{3}k(f(x_1,y_0)+4f(x_1,y_1)+f(x_1,y_2))\right]+$$

$$+\frac{1}{3}k\left[f(x_2,y_0)+4f(x_2,y_1)+f(x_2,y_2)\right]\}=$$

$$=\frac{1}{9}h*k\left[f(x_0,y_0)+4f(x_0,y_1)+f(x_0,y_2)+4f(x_1,y_0)+16f(x_1,y_1)+\right.$$

$$\left.+4f(x_1,y_2)+f(x_2,y_0)+4f(x_2,y_1)+f(x_2,y_2)\right] \tag{9}$$

The form of double integral formula on the rectangle surface area is as follows:

$$\iint_R f(x,y)dx,dy \;=\; \frac{1}{9}h*k\sum_{i=1}^{2n}\sum_{j=1}^{2m}\lambda_{i,j},f_{i,j} \tag{10}$$

where:

R - surface area of rectangle,

h, k – integration steps (in direction X i Y),

$\lambda_{i,j}$ - height multipliers,

$f_{i,j} = f_{(x_i,y_j)}$

where:

$x_i = x_0 + i*h$ *(i=0,1,2,...2n)*

$y_j = y_0 + j*k$ *(j=0,1,2,...2m)*

If we consider irregular surfaces, then these surfaces should be limited within the range of the rectangle. Next we create function, which assumes the same value as function *f(x,y)* in the points on the given surface. As before values are equal to zero in points out of the range of a given surface.

3. Experimental calculation

In (Piasek, 2001) the practical calculation are described, which are based on the fragment of large industrial excavation. The measurement on this area has been performed with the use electronic tacheometer TC - 2003. The obtained numbers of measured points are from 29 to 43 on the area of one hectare. After generation of the net of triangles the mathematical approximation of measured "calculation zones" has been performed by means of the elevation function of the first and of the second degree.

Calculated relative errors δ_{NR} are related to heights HR (analytically calculated), which are assumed to be accurate. Then: values of relative errors δ_{NRmin}=0.00%, δ_{NRmax}=0.04%, and $\delta_{NRaverage}$ for 40 measurement points=0.01%. From ref. [4] results that by application of shape function of the second degree we obtain the increase in approximation accuracy of analysed surface in relation to linear function. The range of increased accuracy is from δ_{LNmin}=0.0% to δ_{LNmax} =0.14%, and $\delta_{LNaverage}$ for 40 measurement points =0.05%.

In considered area (Piasek, 2002/1), calculation of earth mases cubature was carried out in the two "calculation zones". The first surface – layers of excavated earth were represented by so called "basic zones". The second surface is represented by comparative level. In order to carry out analysis for each division of "calculation zones", different number of integration steps *k*h* have been used.

The results of calculation are presented in table 1, where:

VL – calculated cubatures (layers were approximated with linear function),

VN – calculated cubatures (layers were approximated with non - linear function).

Table 1. Relation between number integration steps and accuracy of calculated cubatures

Calculation zone 1

Number of integration steps K*H	Cubatures VN m^3	Cubatures VL m^3	Relative Error $\delta VL\%$
8*8	0.644E+0.6	0.644E+0.6	0.0
10*10	0.676E+0.6	0.675E+0.6	0.14
12*12	0.706E+0.6	0.705E+0.6	0.14
14*14	0.718E+0.6	0.716E+0.6	0.28
16*16	0.728E+0.6	0.727E+0.6	0.14
18*18	0.707E+0.6	0.707E+0.6	0.0
20*20	0.706E+0.6	0.705E+0.6	0.14
22*22	0.715E+0.6	0.714E+0.6	0.14
24*24	0.721E+0.6	0.720E+0.6	0.14
26*26	0.709E+0.6	0.708E+0.6	0.14
28*28	0.716E+0.6	0.715E+0.6	0.14
30*30	0.721E+0.6	0.720E+0.6	0.14
32*32	0.726E+0.6	0.725E+0.6	0.14
34*34	0.711E+0.6	0.711E+0.6	0.0
36*36	0.719E+0.6	0.719E+0.6	0.0

Calculation zone 2

Number of Integration steps K*H	Cubatures VN m^3	Cubatures VL m^3	Relative Error $\delta VL\%$
12*12	0.231E+0.5	0.233E+0.5	0.87
14*14	0.235E+0.5	0.237E+0.5	0.85
16*16	0.241E+0.5	0.243E+0.5	0.83
18*18	0.242E+0.5	0.245E+0.5	1.24
20*20	0.245E+0.5	0.248E+0.5	1.22

On the base of following formula:

$$K*H = \sqrt{number \quad of \quad generated \quad triangle} *2$$

correct number of integration steps is equal to;
- *K*H* = 26*26 for the first calculation zone,
- *K*H* = 16*16 for the second calculation zone,
In consequence correct cubatures are equal to:
- for the first calculation zone VN = 0.243E + 0.5,
- for the second calculation zone VN = 0.708E + 06.

3. Conclusion

Cubatures should be calculated for each layer with the use of non – linear shape approximation function. The main problem involves developments of methods for accurate measurement of layers.

References:

Piasek Z. (2001). Numerical approximation of large industrial excavations. Heidelberg. *AVN*, 8-9, 318-322.

Piasek Z. (2002/1). Irregular net generation in the large industrial excavation. Heidelberg *AVN*,7,247-259.

Piasek Z. (2002/2). Numerical generation of slope surface for large industrial embankments. Heidelberg. *ANV*, 8-9, 274-290.

Service Special des Autoroutes. (1968) – *Calcul Electronique des traces*. Ministere de I Equipement et du Logement. Paris.

Torlegard K., Ostman A. and Lindgren R.A. (1986).Comparative test of photo- *grammetrically sampled Digital Elevation Model*. Department of Photogrammetry, Royal Institute of Technology. Stockholm. S-10044.