GNSS/GPS/LPS based Online Control and Alarm System (GOCA) - Mathematical Models and Technical Realisation of a System for Natural and Geotechnical Deformation Monitoring and Analysis -

Reiner Jäger[1)], Simone Kälber[2)] , Manuel Oswald[1)] and Martin Bertges[3)]

[1)]Hochschule Karlsruhe für Technik und Wirtschaft - University of Applied Sciences Moltkestrasse 30, D-76133 Karlsruhe. Email: reiner.jaeger@goca.info
[2)]D-76227 Karlsruhe. Email: simone.kaelber@web.de.
[3)]Fa. DrBertges Vermessungstechnik, D-66887 Neunkirchen. Email: info@drbertges.de

Abstract: The real-time mobile or permanent multi-sensor system GOCA (GNSS/LPS/LS based Online Control and Alarm System) applies GNSS, terrestrial local positioning sensors (LPS), such as e.g. total stations and leveling instruments, and additionally local sensor (LS), such as e.g. strain-sensors or inclinometers, for a deformation monitoring and analysis. GOCA may be set up as an early-warning-system for natural hazards e.g. landslides, for the monitoring of geotechnical structures, e.g. mining and tunneling activities, and for constructions and buildings monitoring, e.g. dams. The GOCA system consists of GNSS, LPS and LS sensors, which are set up in the monitoring area as a permanent array or as a mobile monitoring system, and the GOCA software components. The first software component is the so-called GOCA hardware-control and communication module, which collects the GNSS, LPS and LS data. Presently the hardware-control and communication software DC3©DrBertges and MONITOR©GeoNav_Trimble are available. They provide the sampled sensor data in a well-defined data-interface format, the so-called GKA format, and provide it to the second software component, the GOCA deformation-analysis software. The GOCA deformation-analysis software is responsible for the further processing of the GNSS- and LPS-data in a three steps sequential adjustment procedure. The 1st step initializes the monitoring reference frame – the coordinates and covariance matrix – consisting of stable reference points. The 2nd and 3rd step comprise the geo-referencing of the three-dimensional object-point coordinates in the reference frame, and the simultaneous deformation analysis. Both least squares and robust techniques (L1-norm and M-estimators) are applied. The deformation analysis step comprises an online-displacement estimation and a Kalman-filtering for the estimation of the object-point state vector of displacements, velocities and accelerations, and additional statistical testing and alarm settings. With respect to the deformation monitoring in geo-techniques and structures the further development of the mathematical model deals with the integration of both, additional parameters (e.g. material parameters, safety-critical damage models) as well as of further, so-called local sensors (e.g. strain-/stress-sensors), by means of a so-called system analysis. Respective FEM-based approaches for static and dynamic processes and the integration of the approaches are treated in the GOCA system context. The mathematical model of the FEM based system analysis approach is appropriate for the evaluation of an optimal sensor configuration on leading the model back to the classical 1st and 2nd order design problem of geodetic network optimization.

1. Introduction

The sensor hardware of the GOCA system may consist of GNSS sensors, terrestrial local positioning sensors (LPS), such as e.g. total stations and leveling instruments and additionally local sensors (LS), such as e.g. strain and stress sensors. As concerns the use of the GOCA-system in practice t is referred to [1], [4], [5], [6], [8], [9] and www.goca.info. The GNSS and LPS sensor data are provided by a hardware-control- and communication module and a respective data interface. The observations **l** derived form that data are used in order to set up a classical geodetic deformation network, as defined in [15] in terms of a permanent online adjustment. A classical geodetic deformation network consists of a stable reference frame $\mathbf{x}_R$, which is set up by the adjusted coordinates of the reference points and moving object-point positions $\mathbf{x}_0(t)$ (fig. 1). This is done in the subsequent 1st and 2nd adjustment step by the GOCA deformation analysis software. By fulfilling the tasks of a classical so-called deformation network the GOCA deformation analysis software gives its first priority to use the adjusted time series $\mathbf{x}_O(t)$ for a further estimation of the state vectors of the three-dimensional displacement, velocity and acceleration $\mathbf{u}_O(t)$, $\dot{\mathbf{u}}_O(t)$ and $\ddot{\mathbf{u}}_O(t)$ of the object points in the deformation analysis, as the 3rd step of the GOCA deformation analysis software. Here the online displacement estimation and the Kalman-filtering are based on least squares or robust estimation procedures, and they are the most relevant and powerful deformation process estimators of the object-point position series $\mathbf{x}_0(t)$ based on GNSS and LPS sensor data.

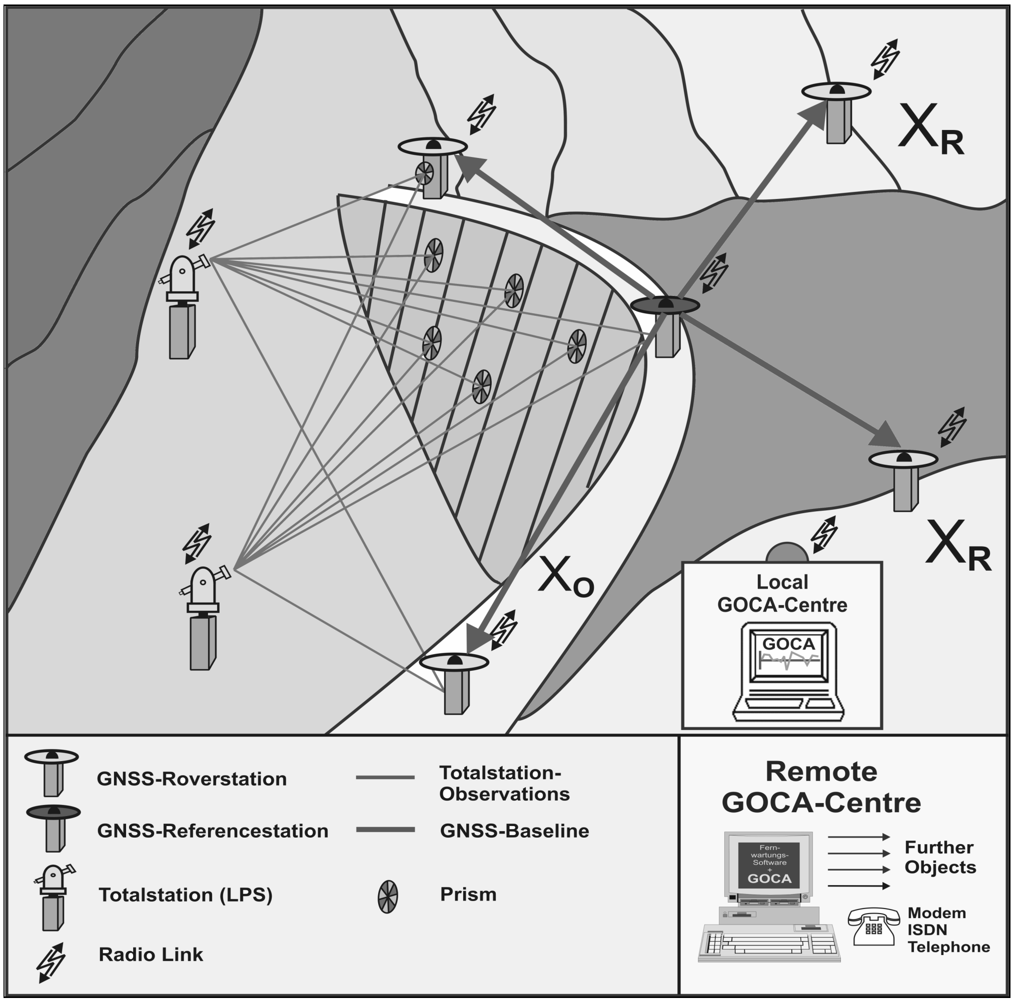

Figure.1: A classical deformation network (stable reference points $\mathbf{x}_R$ and moving object points $\mathbf{x}_O(t)$ set up online by using GNSS and LPS sensor data in the GOCA system. With the GOCA software several local objects can be monitored simultaneously in a unified reference frame.

The observation data **l**(t) of the different LS sensor types can also be treated within the displacement-estimation and Kalman-filtering in the 3rd deformation analysis step. In that case the displacement, velocity and acceleration quantities are directly related to the state vector of the sensor data **l**(t), e.g. the local inclinations. The system analysis related deformation analysis however is appropriate to lead all sensor data (GNSS, LPS and LS) back to a common parametric adjustment model, which contains both the displacement information **u** as well as additionally the physical parameters **p** of an object.

2. Data flow and deformation analysis processing steps in the GOCA system

The data flow and data interfaces of GOCA system are shown in fig. 2. The hardware-control and -communication software is responsible for the sensor control and data communication in the sensor array of a GOCA-system. The selection of the sensor types depends on the dynamics and amplitude of the expected object point changes $\mathbf{x}_O(t)$ in time t. In accordance with the sampling theorem a sampling rate is chosen, which corresponds to the required deformation spectrum. The questions of sampling, re-sampling and discretization are regarded in [2]. The hardware-control-software finally provides the sensor observations with a time stamp on every dataset in the well-defined and open GKA data format [14], which is similar to NMEA. The GKA data is organized in different sensor message types. The structure of the GKA-data messages is adapted to the GNSS baseline- and session output, the LPS data standard observables (zenith angles, slope distances, directions, leveled height differences), and the output of different types of local sensors (LS), e.g. strain- and stress-sensors.

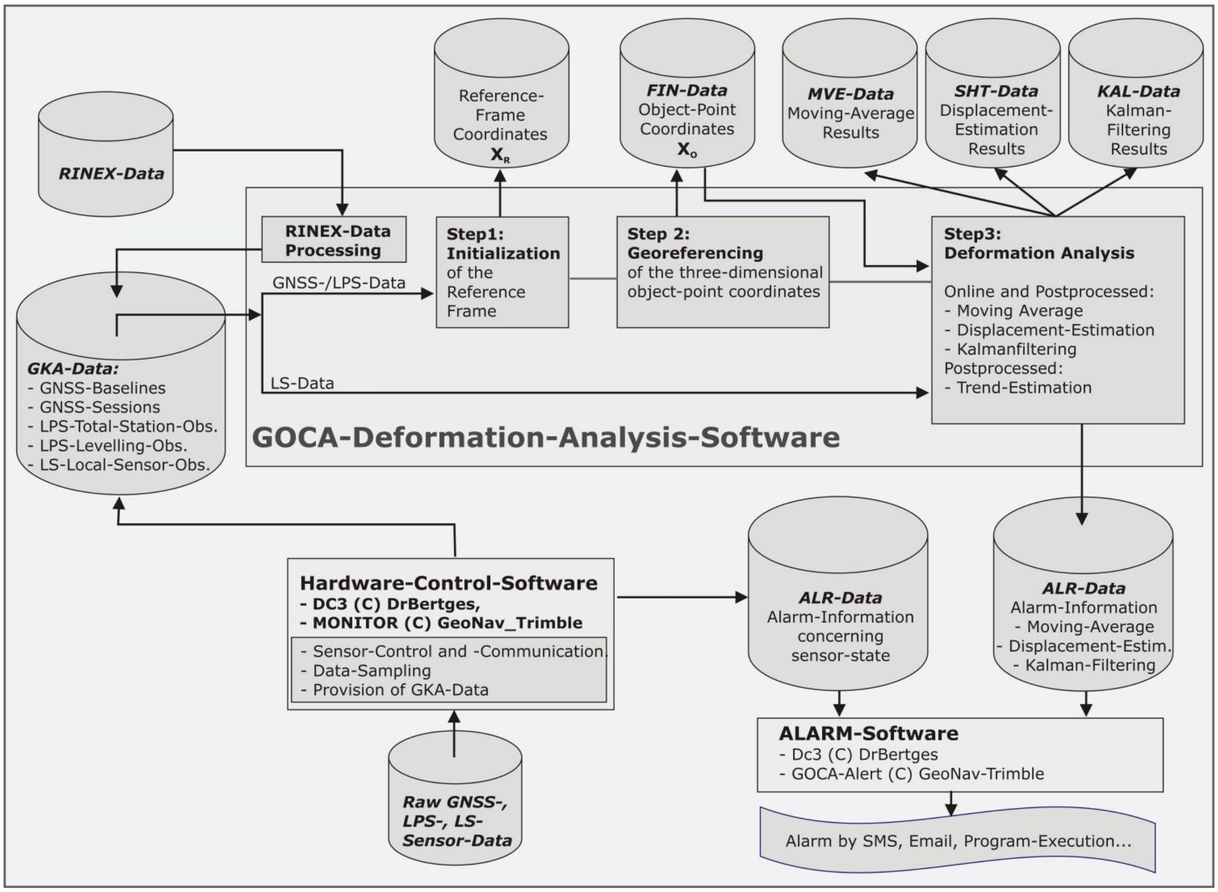

Figure 2: Data flow and deformation analysis processing steps in the GOCA system

In the case of near-online processing projects also GNSS RINEX-files can be taken from the GNSS-sensors and transmitted to the GOCA deformation analysis software to be processed with different GNSS-processing engines. The RINEX-processing results are leaded again back to the GKA format. The RINEX data transfer can also be managed by the hardware-control-software. Presently two different packages, namely MONITOR of GeoNav_Trimble and S+H Systemtechnik (www.sh-systemtechnik.de) and GOCA_DC3 of the company DrBertges Vermessungstechnik (www.drbertges.de) are commercially available. These hardware-control-software packages are completed by alarm-software modules. These handle both the type of alarm messages in kind of sensor failures and the alarm information (ALR-files), which result from the different deformation estimations statistical tests used for the detection of critical deformation states of the object (fig. 1).

A first task of the GOCA deformation analysis software is to evaluate the parameter estimation of a classical deformation network ([1], [13], [15]) online. In case of GNSS- and/or LPS sensor data the adjustment of the respective GKA data provides the adjusted absolute object-point-positions $\mathbf{x}_O(t)$ in the 2nd step of the deformation analysis procedure. The reference frame $\mathbf{x}_R$ is estimated in the 1st step. The results $\mathbf{x}_O(t)$ and the covariance matrices $\mathbf{C}_{x,O}(t)$ are stored in so-called FIN-files, and the moving average estimation result is stored in MVE-files [14]. By the estimation of the object-point displacements $\mathbf{u}_O(t)$, velocities $\dot{\mathbf{u}}_O(t)$ and accelerations $\ddot{\mathbf{u}}_O(t)$ in the 3rd step the GOCA system fulfils the second task of a classical geodetic deformation analysis. The respective online displacement and the Kalman-filtering estimation both use the output of th second step - the time series $\mathbf{x}_O(t)$ - as observation input. An analogous displacement, velocity and acceleration estimation with the same algorithms is based on the on the GKA-data $\mathbf{l}(t)$ of the local sensors (LS) as observations and leads to the estimations $\Delta\mathbf{l}_O(t)$, $\Delta\dot{\mathbf{l}}_O(t)$ and $\Delta\ddot{\mathbf{l}}_O(t)$. The results of the displacement and the Kalman-filter estimations are stored in so-called SHT- and KAL-files respectively. The final task of the deformation analysis procedure in the 3rd step is to set up an alarm in case of the detection of significant critical parameters $\mathbf{u}_O(t)$, $\dot{\mathbf{u}}_O(t)$, $\ddot{\mathbf{u}}_O(t)$ as well as for the state vectors $\Delta\mathbf{l}_O(t)$, $\Delta\dot{\mathbf{l}}_O(t)$, $\Delta\ddot{\mathbf{l}}_O(t)$ concerning the state vector changes of the LS array. The alarm messages are stored in so-called ALR-files. The GOCA deformation analysis software archivates the above FIN-, MVE-, SHT-, KAL and ALR-files in a project-related directory stucture. These data can be visualized in the graphics representation of the GOCA deformation analysis software, and above this it is provided as open interface for external further processing, e.g. for web-based representations, the use in GIS and last but not least, for system-analysis related deformation approaches (chap. 4).

3. GOCA Deformation Analysis Software

3.1. Mathematical Modelling for GNSS- and/or LPS-Data and for LS-data

As shown in fig. 2, the 3rd step of the subsequent adjustments of the GOCA deformation analysis software procedure serves as deformation parameters estimation step. Observations for classical geodetic displacement estimations is the position data ($\mathbf{x}_O(t)$, $\mathbf{C}_{x,O}(t)$) of the object-points geo-referenced in the frame $\mathbf{x}_R$ (fig. 1) That means that the subsequent 1st and 2nd adjustment steps constitute the heart of the GOCA software with respect to the online modelling of a classical geodetic deformation network providing absolute object-point positions and movements using GNSS and LPS data. Independently the GKA-data ($\mathbf{l}(t)$, $\mathbf{C}_l(t)$) of local sensors (LS) can be processed by the displacement and Kalman-filter algorithms of the 3rd step, the so called geometrical deformation analysis ([1], [13], [16]). The FEM-based system analysis approach presented in chap. 4 is appropriate to parametrize commonly all these sensor observation types, namely GNSS, LPS (including the LPS category of terrestrial laser-scanners) and LS sensors.

3.1.1. First and Second Adjustment Step – Reference and Object Point Coordinate and Height Surface Transition Estimations

The deformation analysis concept implemented in the GOCA software is due to a classical geodetic deformation analysis. That means, that for two different observation epochs t_i and t_j, we get the following system of observation equations:

$$\mathbf{l}(t_i) + \mathbf{v}_i = \mathbf{A}_{Ri} \cdot \mathbf{x}_{Ri} + \mathbf{A}_{Oi} \cdot \mathbf{x}_O(t_i) \text{ and } \mathbf{C}_{li} , \tag{1a}$$

$$\mathbf{l}(t_j) + \mathbf{v}_j = \mathbf{A}_{Rj} \cdot \mathbf{x}_{Rj} + \mathbf{A}_{Oj} \cdot \mathbf{x}_O(t_j) \text{ and } \mathbf{C}_{lj} . \tag{1b}$$

With $\mathbf{A}$ we describe the design matrices of the linearized Gauß-Markov model, with $\mathbf{l}$ the GNSS- and LPS-based observations and with $\mathbf{C}_l$ their stochastical models. The stability of the reference frame is modelled on introducing the same coordinates $\mathbf{x}_{Ri} = \mathbf{x}_{Rj} = \mathbf{x}_R$ for all epochs, while for the object-points different coordinates $\mathbf{x}_{Oi}$ and $\mathbf{x}_{Oj}$ are estimated for different epochs t_i and t_j, and the stability is to be checked based on (1a,b) by strict statistical concept as described in [16]. With respect to a discretization in time, the "epochs" t_i and t_j are extended to time intervals of length ΔT. The adjustment intervals ΔT has to be equal or larger than the sensor data sampling interval Δt. The epoch time stamps t_i and t_j are accordingly the centres of the subsequent intervals ΔT.

The GNSS- and LPS-based observations $\mathbf{l}$ in (1a, b) are derived from the original GNSS and LPS GKA-data in such a way, that a separate plan and height adjustment is to be performed at each epoch based on the data in the interval ΔT. This has the advantage that the adjustment model (1a,b) can easily be handled in the case, that the used sensor type in a GOCA-array, such as pure hydrostatical levels, enables for example only a vertical monitoring. The observation quantities $\mathbf{l}$ derived form GNSS GKA-data and used in (1a,b) are the 2D/1D-baseline observations $(\Delta x, \Delta y \mid \Delta h)_{ij}$. The observation quantities $\mathbf{l}$ derived from LPS data and used in (1a,b) are the plan distances s_{ij}, the directions r_{ij} and the height differences ΔH_{ij}. So the partly linear and non-linear observation equations (1a,b) for an epoch t_i read as follows [7]:

$$\begin{bmatrix} \Delta x_{ij} \\ \Delta y_{ij} \end{bmatrix}_{GNSS} + \begin{bmatrix} v_{\Delta x,ij} \\ v_{\Delta y,ij} \end{bmatrix}_{GNSS} = \begin{bmatrix} \Delta\hat{x}_{ij} \\ \Delta\hat{y}_{ij} \end{bmatrix} \quad , \tag{2a}$$

$$s_{ij} + v_{s,ij} = s \cdot \sqrt{\Delta\hat{x}_{ij}^2 + \Delta\hat{y}_{ij}^2} \quad , \tag{2b}$$

$$r_{ij} + v_{r,ij} = \arctan\left(\frac{\Delta y_{ij}}{\Delta x_{ij}}\right) - o_i \quad , \tag{2c}$$

$$\Delta h_{GNSS,ij} + v_{\Delta h,ij} = \Delta\hat{h}_{ij} \qquad \text{and} \qquad 2d)$$

$$\Delta H_{terr,ij} + v_{\Delta H,ij} = s_h \cdot \Delta\hat{h}_{ij} + (\hat{a}_{00} + \hat{a}_{10} \cdot x_j + \hat{a}_{01} \cdot y_i)^m - (\hat{a}_{00} + \hat{a}_{10} \cdot x_j + \hat{a}_{01} \cdot y_i)^n \quad . \tag{2e}$$

Besides the coordinate unknowns $(\hat{x}, \hat{y} \mid \hat{h})$ of the reference points $\mathbf{x}_R$ and the object-points $\mathbf{x}_0$ (fig. 1), which are set up in the GNSS-frame, the scale factors s and s_h for the plan and height component, the orientation unknown o_i for the direction measurements and the polynomial parameters $\hat{\mathbf{p}}_N = (\hat{a}_{00}, \hat{a}_{10}, \hat{a}_{01}, ...)^T$ occur as additional auxiliary parameters. The parameters $\hat{\mathbf{p}}_N$ model the height reference surface ("geoid") in the local object area (see www.dfhbf.de). So the monitoring of several local objects in an extended but unique reference frame $\mathbf{x}_R$ is enabled by different parameter sets $\hat{\mathbf{p}}_N^m$, $\hat{\mathbf{p}}_N^n$ etc. . These are defined by the deformation network design settings in the GOCA deformation analysis software. The initialization, namely the 1st adjustment step, is based on a least squares (L2-norm) free network adjustment of the GNSS- and LPS-based observations (2a-e) data with respect to a user-defined starting epoch and initialization time interval. The initialization is robustified with respect to gross errors by an automatic iterative data snooping, including an iterative variance-component estimation. By the aim of realizing a classical deformation network analysis online, this 1st step has to precede the deformation monitoring, as it provides the network datum and frame $\mathbf{x}_R$ (fig.1). The 2nd step in GOCA deformation analysis software is again related to (1a, b) with respect to different extended epochs t_i and t_j, and is running online using the observation equations (2a-e) for the GNSS- and LPS data within the epochs of duration ΔT . This 2nd step comprises the permanent adjustment of the GNSS and LPS data and provides the three-dimensional georeferencing of the object-point positions $\mathbf{x}_O(t)$ (3a). The estimated adjusted object-point positions $\mathbf{x}_O(t)$ and the covariance matrix $\mathbf{C}_O(t)$ are stored in daily FIN-files (fig.2). The reference frame coordinates $\mathbf{x}_R$ and the above auxiliary parameters (except the orientation-unknowns o_i) are kept a fix parameters in the 2nd step according to the results of the 1st step. The covariance matrix of these parameters is however strictly considered in the computation of $\mathbf{C}_O(t)$ [16].

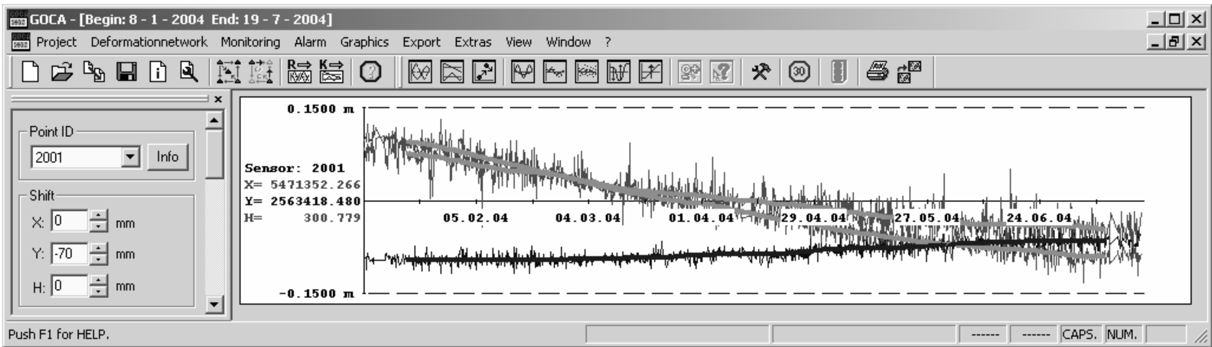

Figure 3: GOCA object-point time series $\mathbf{x}_O(t_i)$ as result of the 2nd adjustment step. The thick lines show the smoothing by a moving average estimation (MVE).

3.1.2. Third Adjustment Step – Deformation Analysis

The 3rd step, namely the deformation analysis, deals with the estimation of the parameters of different deformation functions, and it runs online in parallel to the 2nd step. The deformation parameter estimation is related to the object-point position time-series and their covariance matrix, reading

$$\mathbf{x}_O(t) \text{ and } \mathbf{C}_O(t) \qquad . \qquad . \qquad \text{(3a, b)}$$

These are used as observations for the parameter estimation in step 3. As a first and simple object-point related deformation function the GOCA deformation analysis software provides a moving average estimation (MVE) including the detection of critical displacements (fig. 3). A second deformation function is the online displacement estimation ([9], [14]) between different "extended epochs" t_0 and t_i . Extended epochs means again, that the two epochs t_0 and t_i start at individual times t_0 and t_i , and have interval lengths ΔT_o and ΔT_i (fig. 4), e.g. one hour. The start of the first epoch t_0 may be the initialization time (1st step). Alternatively t_0 can be defined in the GOCA deformation analysis software settings (fig. 4). either by an arbitrary fixed time mark, or by a dynamically moving time mark. The functional model of the object point displacement estimation reads:

$$\begin{bmatrix} \mathbf{l}_{t_0} \\ \mathbf{l}_{t_i} \end{bmatrix} + \begin{bmatrix} \mathbf{v}_{t_0} \\ \mathbf{v}_{t_i} \end{bmatrix} = \begin{bmatrix} \mathbf{E}_1 & \mathbf{0} \\ \mathbf{E}_2 & \mathbf{E}_2 \end{bmatrix} \cdot \begin{bmatrix} \hat{\mathbf{x}}_0 \\ \hat{\mathbf{u}}(t) \end{bmatrix} = \mathbf{A} \cdot \hat{\mathbf{y}} \quad \text{with} \quad \hat{\mathbf{y}} = [\hat{\mathbf{x}}_0(t_0), \hat{\mathbf{u}}(t_0, t_i)]^T \quad . \qquad \text{(4a, b)}$$

The two observation groups $\mathbf{l}_{t_0}$ and $\mathbf{l}_{t_i}$ and their covariance matrices are taken out of the object-point time series information (3a,b). With $\mathbf{v}$ we introduce the observation corrections. To mark the difference between the time series observations $\mathbf{x}_O(t)$ (3a) and the estimated epoch state $\hat{\mathbf{x}}_O(t_0)$ we introduce in (4a,b) the sign (^) for the estimated deformation parameters. The six deformation parameters included in $\hat{\mathbf{y}}(t)$ are for each object point the three-dimensional adjusted epoch state position $\hat{\mathbf{x}}_0 = [\hat{x}, \hat{y} \mid \hat{h}]^T_{t_0}$ at the reference time t_0, and the three-dimensional displacements $\hat{\mathbf{u}}(t_0, t_i) = [u_x, u_y \mid u_h]^T_{t_0,t_i}$ dating to the second epoch starting at time t_i. The design matrices $\mathbf{E}_1$

and $\mathbf{E}_2$ are column matrices composed of (3 x 3)-unit matrices for each three-dimensional point observation $\mathbf{x}_O(t)$ in the respective epoch intervals ΔT_0 and ΔT_i. Fig. 4 shows the GOCA-software settings for the online displacement estimation according to (4a, b). The different settings concern the selection of the object points, the epoch definition for the displacement estimation, the settings for adjustment and statistical testing, and the settings for an automatic alert.

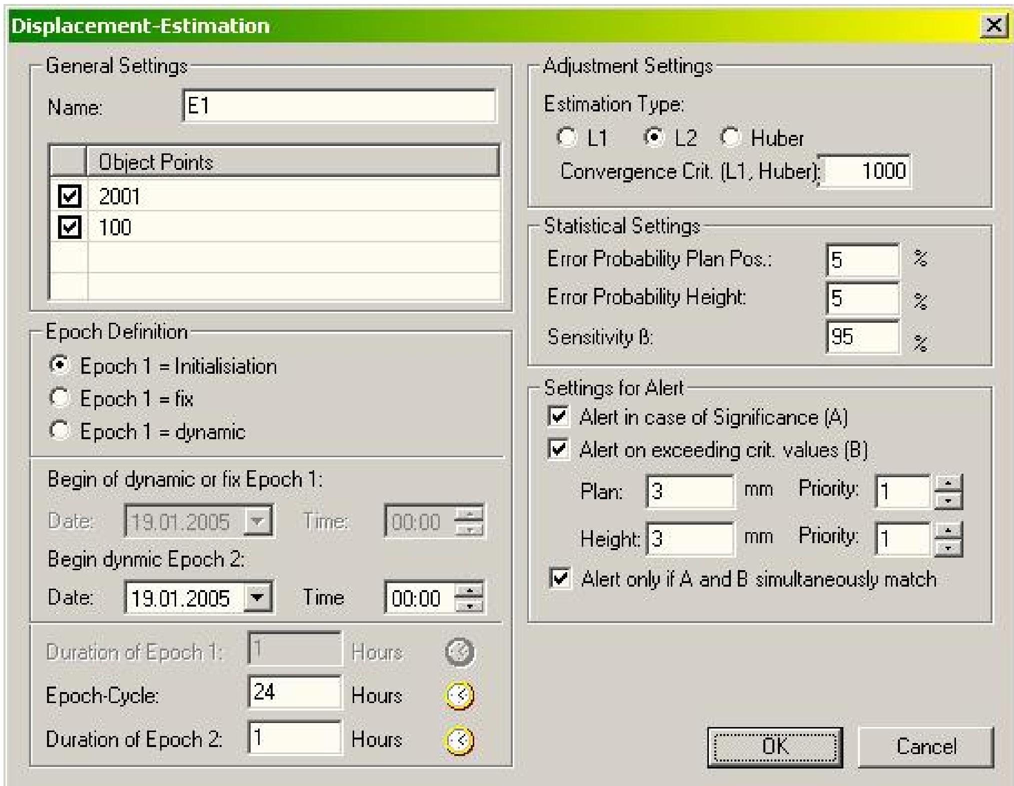

Figure 4: Settings for the GOCA online displacement estimation and alerting.

The GOCA Kalman-Filtering as the third component for the deformation parameter estimation in the 3rd step is related to the so-called transition equation (5a) and to the state vector **y**(t), reading:

$$\begin{bmatrix} \mathbf{u}(t) \\ \dot{\mathbf{u}}(t) \\ \ddot{\mathbf{u}}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{I} & [\Delta t] & \left[\frac{1}{2}\Delta t^2\right] \\ \mathbf{0} & \mathbf{I} & [\Delta t] \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{u}(t-\Delta t) \\ \dot{\mathbf{u}}(t-\Delta t) \\ \ddot{\mathbf{u}}(t-\Delta t) \end{bmatrix} \quad \text{and} \quad \mathbf{y}(t) = [\mathbf{u}(t), \dot{\mathbf{u}}(t), \ddot{\mathbf{u}}(t)]^T \quad . \qquad (5a,b)$$

The state vector **y**(t) of the GOCA-Kalman-Filtering comprises the individual three-dimensional displacements $\mathbf{u}(t)$, the velocities $\dot{\mathbf{u}}(t)$ and the accelerations $\ddot{\mathbf{u}}(t)$ of the object points between subsequent time intervals Δt. The observations **l**(t) and their covariance matrices for the Kalman-filtering (5a,b) are again set up from the object point time series (3a,b) as

$$\mathbf{l}_u(t) = \mathbf{x}_O(t) - \mathbf{x}_O(t_0) \quad . \qquad (5c)$$

Again t_0 and t are "extended epochs", while t_0 is a fixed time mark in the Kalman-filtering. The principles and concepts of the significance tests (fig. 4, right) for of the deformation parameters are treated in [1], [2] and [16].

3.2. Geometrical Deformation Analysis for LS -Data

Local sensor data $\mathbf{l}(t)$, e.g. strain-, stress-sensors and inclinometers and the covariance matrix information $\mathbf{C}_l(t)$ can be used in the moving average estimation (MVE) as well as in the sophisticated models of the online displacement estimation (4a,b) and in the Kalman-filtering (5a,b) in GOCA, in analogy to the observation information and models for the absolute object-point positions (3a,b).

3.3. Estimation principles and Relevance of Online Monitoring Systems

The parameter estimation principle in the GOCA deformation analysis software is based on the general concept of an M-estimation [7] with an estimation function $\rho(\bar{v}_k)$. It reads:

$$\sum_{k=1}^{n}\rho(\bar{v}_k) = = \sum_{k=1}^{n}\rho\left((\mathbf{C}_l^{-\frac{1}{2}}\cdot\mathbf{A})_k\cdot d\hat{\mathbf{y}} - (\mathbf{C}_l^{-\frac{1}{2}}\cdot(\mathbf{l}-\mathbf{l}(\mathbf{y}^0)))_k \right) = \text{Min}\,|_{\hat{\mathbf{y}}} \quad . \tag{6}$$

Depending on estimation function $\rho(\bar{v}_k)$, the estimated parameters $\hat{\mathbf{y}}$ are received due to a least squares estimation with $\rho(\bar{v}_i) = \frac{1}{2}\bar{v}_i^{\,2}$, a robust L1-estimation with $\rho(\bar{v}_i) = \frac{1}{2}|\bar{v}_i|$, and a weakly robust Huber-estimation with $\rho(\bar{v}_i) = \begin{cases}\frac{1}{2}\bar{v}_i^2 & \forall\ |\bar{v}_i| \le k \\ |\bar{v}_i| & \forall\ |\bar{v}_i| > k\end{cases}$ (fig. 4, right). The L1 norm and the Huber-estimation are robust against the occurrence of gross observation errors $\nabla\mathbf{l}$. As it is a statistical fact that mor or less 0.3% of the observations are concerned with gross errors, the availability of a robust parameter estimation related to (6) and adequate robust estimation functions $\rho(\bar{v}_k)$ increases essentially the reliability of an online monitoring system like GOCA.

4. System Analysis – General Aspects and Contributions of GOCA

The further development of the deformation analysis clearly shows the trend to integrate the results of a geodetic displacement estimation (4a,b) and/or Kalman-filtering (5a,b,c), namely $\mathbf{u}(t)$, $\dot{\mathbf{u}}(t)$ and $\ddot{\mathbf{u}}(t)$ as well as geometrical or physical observations $\mathbf{l}(t)$ of local sensors (LS) into a common modeling ([1], [3], [11], [13], [16]). With the property that both deformation function types are observed output signals of the physical object state a physical kind of model is needed for this integration. Within the classification into black- grey- and white-box models, the class of Finite Element Models (FEM) belong to the white-box category and is parametrized both by physical parameters $\mathbf{p}$ and by the displacements state vectors $\mathbf{u}(t)$, $\dot{\mathbf{u}}(t)$ and $\ddot{\mathbf{u}}(t)$ as the parameters of a so-classical so-called geometrical deformation analysis ([13],[16]). So FEM are the key for an integrated modeling of geodetic displacements $\mathbf{u}$ evaluated online from GNSS- and LPS-sensors and LS observations $\mathbf{l}$. The FEM system equations then can be written as

$$\mathbf{F}(\mathbf{p},\mathbf{u}(t),\dot{\mathbf{u}}(t),\ddot{\mathbf{u}}(t)) = \mathbf{0} \tag{7a}$$

While in the dynamic case - e.g. the FEM of vibrating structures - the derivatives $\dot{\mathbf{u}}(t)$ and $\ddot{\mathbf{u}}(t)$ are relevant ([3], [16]), the state vectors $\dot{\mathbf{u}}(t)$ and $\ddot{\mathbf{u}}(t)$ do not occur in the so-called static case ($\dot{\mathbf{u}} \rightarrow \mathbf{0}$). The number of parameters $\mathbf{p}$ and system matrices also decreases, and we arrive at the FEM system equation of type $\mathbf{F}(\mathbf{p},\mathbf{u}) = \mathbf{0}$. It reads

$$\mathbf{K}(\mathbf{p}_k)\cdot\mathbf{u} - \mathbf{f} = \mathbf{0} \quad . \tag{7b}$$

in the case of the FEM of an elastic structure, e.g. dams ([11], [16], [18], fig.1). The characteristic system matrix is the so-called stiffness matrix $\mathbf{K}(\mathbf{p}_k)$, $\mathbf{f}$ is the vector of external nodal-forces, and $\mathbf{p}_k$ is the vector of material-parameters of the FEM elements ([10], [11], [12], [16], [18], fig. 5)). As extension of the FEM approach presented [11], we introduce in the following system analysis related adjustment approach, by $\Delta\hat{\mathbf{p}}_k$, an additional set of unknown FEM parameters into the system equations. The parameters $\Delta\hat{\mathbf{p}}_k$ model that kind of safety-critical parameters (e.g. local failures of the structure, such as a washout in case of an earth dam, or fissures), which are to be detected by statistical methods a part of the monitoring system. The other FEM parameters $\mathbf{p}_k$ and $\mathbf{f}$ are known and introduced as direct observations, and their uncertainties are put into respective covariance matrices. So the FEM of an object - as first part of the mathematical adjustment model representing the system analysis approach – reads in the static case:

$$\mathbf{0}_{sys} + \mathbf{v}_{sys} = \hat{\mathbf{u}} - \mathbf{K}(\hat{\mathbf{p}}_k, \Delta\hat{\mathbf{p}}) \cdot \hat{\mathbf{f}} \quad \text{and } \mathbf{C}_{sys} \to \mathbf{0} \quad , \tag{8a}$$

$$\mathbf{p}_k + \mathbf{v}_k = \hat{\mathbf{p}}_k \quad \text{and } \mathbf{C}_{p_k} \quad , \tag{8b}$$

$$\mathbf{f} + \mathbf{v}_f = \hat{\mathbf{f}} \quad \text{and } \mathbf{C}_f \quad . \tag{8c}$$

As concerns the parametric FEM representation in the case of structural dynamics case, it is referred to [16], [12], [3] and [18]. With $\mathbf{C}_{sys} \to \mathbf{0}$ (8a) the system equation (7b) is set up as condition equation. Alternatively (7b) could also be set up in a Kalman-filter mode together the initial observations (8a,c), whereas the GNSS, LPS and LS observations (8d-e) contribute during the online monitoring as observations in each filtering step. The vector $\hat{\mathbf{u}}$ of the nodal-point displacements concerns the FEM, and so all nodes at the objects on its surface and in its interior (fig. 5).

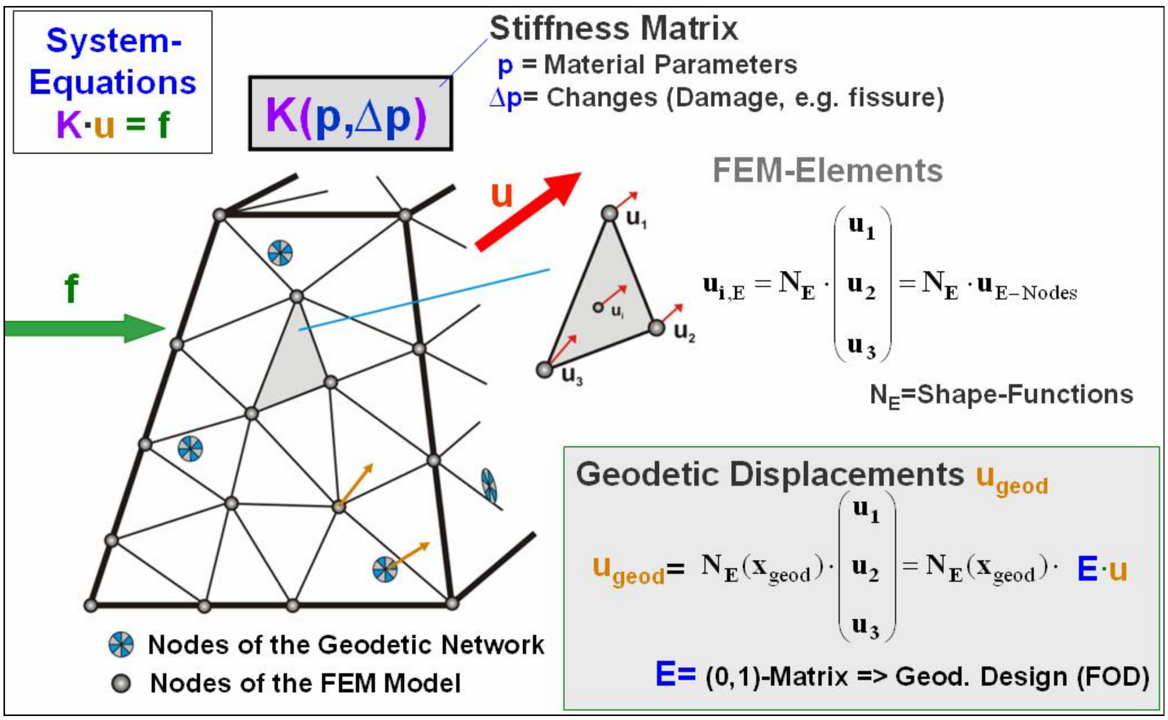

Figure 5: Finite Element Model of a Dam in the static case

Concepts for the statistical testing of the significance of the unknown parameters $\Delta\hat{\mathbf{p}}_k$ - similar to data-snooping – are discussed in [16] and [17]. The geodetic online displacements determined by the GOCA system estimations (4a,b) or (5a,b,c) based on GNSS- and LPS-sensors (chap. 3.1.2) are named here $\mathbf{u}_{GOCA}$ and introduced as the second component of the system analysis model reading

$$\mathbf{u}_{GOCA} + \mathbf{v}_{GOCA} = \mathbf{N}_E(x,y,z) \cdot \mathbf{E}_{GNSS/LPS} \cdot \hat{\mathbf{u}} \quad \text{and} \quad \mathbf{C}_u \quad . \tag{8d}$$

The rectangular matrix $\mathbf{E}_{GNSS/LPS}$ relates the total vector $\hat{\mathbf{u}}$ of the FEM to the three absolute GNSS-/LPS-sensor displacements $\mathbf{u}_{GOCA}$. The matrix $\mathbf{E}_{GNSS/LPS}$ is occupied with “1” at the positions of that nodal point element (fig. 5), which carries the GNSS/LPS sensor, else with “0”. The known so-called shape-function $\mathbf{N}_E(x,y,z)$ ([12], [18]) relates the nodal point displacements of the local (surface) element (fig.5, right) to the elements inside the FEM. The equation (8d) can also be used with respect to position and displacements modelled from on an extension of (2a-e) to laser-scanner data (laser-coordinates and additional transformation parameters, in principle also or raw-data). Finally the "displacement" observations $\Delta\mathbf{l}_O(t)$ from the estimated state vectors (chap. 3.2) of local sensors (LS) are introduced into the system analysis approach. In the case of the so-called geometric type the observation equations read

$$\mathbf{l}_g + \mathbf{v}_g = \mathbf{l}_g(\mathbf{N}_E, \mathbf{E}_g, \hat{\mathbf{u}}) \qquad \text{and } \mathbf{C}_{lg} \quad . \tag{8e1}$$

Example - local strain $\boldsymbol{\varepsilon}$ sensor: $\boldsymbol{\varepsilon} + \mathbf{v}_\varepsilon = \mathbf{L} \cdot \mathbf{N}_E \cdot \mathbf{E}_g \cdot \hat{\mathbf{u}}$. (8e2)

In case of LS of the so-called "physical type" we have

$$\mathbf{l}_p + \mathbf{v}_p = \mathbf{l}_p(\mathbf{N}_E, \mathbf{E}_p, \hat{\mathbf{u}}) \qquad \text{and } \mathbf{C}_{lp} \tag{8f1}$$

Example - local stress $\boldsymbol{\sigma}$ sensor: $\boldsymbol{\sigma} = \mathbf{D}(\mathbf{p}_k) \cdot \mathbf{L} \cdot \mathbf{N}_e \cdot \mathbf{E}_p \cdot \hat{\mathbf{u}}$. (8f2)

The matrices $\mathbf{E}_g$ and $\mathbf{E}_p$ are defined ans set up in analogy to $\mathbf{E}_{GNSS/LPS}$. With $\mathbf{N}_E(x,y,z)$ we introduce again the above shape functions of the FEM element model. The matrices $\mathbf{L}$ and $\mathbf{D}(\mathbf{p}_k)$ comprise known differential operators, and the known element stiffness paramtrization ([10], [11], [12], [18]).

The different local sensors (LS) can be located both on exterior and interior surface and even inside the objects structure, while GNSS- and LPS-sensors are in general located at the exterior and/or interior object boundaries. In this context, the mathematical model (8a-f) of the system analysis based adjustment approach, enables and urges the application of the classical geodetic network optimization principles. Based on the target function of an optimal sensitivity [17] for safety critical parameters $\Delta\mathbf{p}_k$, the matrices $\mathbf{E}_{GNSS/LPS}$, $\mathbf{E}_g$ and $\mathbf{E}_p$ describe the placement of a given number of different sensors types. So the problem of optimum positions for a number of "1" in $\mathbf{E}_{GNSS/LPS}$, $\mathbf{E}_g$ and $\mathbf{E}_p$ constitutes the 1st order design problem of optimal sensor positions in geodetic monitoring networks and systems, and the occupation of the sensor covariance matrix the respective 2nd order design problem of optimal accuracies of the sensor-types.

5. Conclusions

The 3 step network adjustment concept behind the GOCA deformation-analysis software provides a unique geo-refercencing of the GNSS/LPS-occupied object point positions in the coordinate frame of the reference points. The geodetic online deformation analysis of the object-point can be set up as flexible user-defined displacement estimation or by a Kalman filter (displacements, velocities and accelerations). Local sensor (LS) data can also be monitored by the GOCA-system in the 3rd step. Both least squares and robust estimation techniques are applied in the deformation analysis procedures, so that a reliable setting of an alarm is enabled, in case that a critical state is reached and proved by statistical testing. The evaluation of continuous time series of the objects displacement field and of the state vector of local sensors, provided by GOCA, opens new perspectives in deformation analysis and model calibration. This concerns the presented transition from the classical geometric deformation analysis to so-called system analysis based approaches, as required by the interests of geodesists and of other disciplines such as geodynamics, geotechnics and civil engineering. The presented system analysis approach is also open for the 1st and 2nd order design question of an optimum GNSS-/LPS and LS sensor location and precision with the target function of an optimal sensitivity for safety critical parameters.

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