UNCERTAINTY IN TACHEOMETRIC MEASUREMENT OF CONVERGENCES IN TUNNELS

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Abstract: The variations, along time, of the distances between object points on tunnel cross-sections (convergences) may be used in real time continuous tunnel monitoring systems, as decision tools, to activate signals of alert or alarm. Electronic tacheometers, especially the motorized ones, are frequently used to measure convergences. To quantify the levels of alert and alarm one should take into account the convergence measurement uncertainty. The paper deals with modelling the uncertainty of the convergence as a function of the instrumental uncertainty parameters, using the general formula for the propagation of variances. The model is applied to a simulated tunnel, where the special case of motorized tacheometers with automatic target recognition (ATR) systems is analysed.

1. Introduction

One of the most usual applications of tacheometry in tunnels is the monitoring, along time, of variations of distances (convergences) between object points on cross-sections. Sometimes, especially in continuous monitoring systems, the convergences provide early warning of potentially damaging movements and, therefore, are used to activate signals of alert and alarm. For this reason, is very important to estimate the uncertainty of the convergences to quantify realistic levels of alert and alarm since is not advisable to considerer, as significant, convergences smaller than three standard deviations. The paper deals with modelling the uncertainty of convergence as a function of the instrumental uncertainty parameters, using the general formula for the propagation of variances, and will be applied to a simulated tunnel where tacheometers with automatic target recognition (ATR) systems were used.

2. Modelling the Uncertainty

2.1. The Uncertainty of the Coordinates

A tacheometer defines a three-dimensional cartesian referential with the origin in its geometric center, with the zz axis coincident with the direction of the plumb line in the station and with the other two axis in the horizontal plan of the station. We will assume that the zero of the horizontal circle is oriented in the direction transversal to the tunnel (yy axis), being the xx axis oriented in the longitudinal direction (Figures 3 and 4).

Given a point (object point) in a cross-section of the tunnel, materialized by an EDM retro-reflector, the tacheometer measures:

i) horizontal direction, defined by the geometric center of tacheometer and by the point;

ii) zenith direction, defined by the geometric center of tacheometer and by the point;

iii) distance between the geometric center of tacheometer and the point (geometric center of the prism).

The horizontal direction (H_z), the zenith direction (Z) and the distance (S), are spherical coordinates of the object point, associated to the instrumental referential of the tacheometer. In this referential, the cartesian coordinates (x, y, z) of the object point can be computed from the spherical coordinates (Hz, Z, S), by the vector field:

$$\text{i) } x = S \sin Z \sin Hz, \quad \text{ii) } y = S \sin Z \cos Hz, \quad \text{iii) } z = S \cos Z \tag{1}$$

In a reciprocal way, spherical coordinates (Hz, Z, S) can be computed from the cartesian coordinates (x, y, z) of the object point, by:

$$\text{i) } Hz = \arctan\left(\frac{x}{y}\right), \quad \text{ii) } Z = \frac{\pi}{2} - \arctan\left(\frac{z}{\sqrt{x^2+y^2}}\right), \quad \text{iii) } S = \sqrt{x^2+y^2+z^2} \tag{2}$$

If the errors that affect the observations (Hz, Z, S) are modelled with normal distributions, stochastically independent, and with standard deviations σ_{Hz}, σ_Z and σ_S, respectively, the formula of the propagation of the variances allows to establish the matrix of variance of the coordinates (Σ):

$$\Sigma = \begin{bmatrix} \sigma_x^2 & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_y^2 & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_z^2 \end{bmatrix} = J \times \begin{bmatrix} \sigma_{Hz}^2 & 0 & 0 \\ 0 & \sigma_Z^2 & 0 \\ 0 & 0 & \sigma_S^2 \end{bmatrix} \times J^{\mathrm{T}} \tag{3}$$

where J is the Jacobian matrix of the vector field (1):

$$J = \begin{bmatrix} \frac{\partial x}{\partial Hz} & \frac{\partial x}{\partial Z} & \frac{\partial x}{\partial S} \\ \frac{\partial y}{\partial Hz} & \frac{\partial y}{\partial Z} & \frac{\partial y}{\partial S} \\ \frac{\partial z}{\partial Hz} & \frac{\partial z}{\partial Z} & \frac{\partial z}{\partial S} \end{bmatrix} = \begin{bmatrix} +S \sin Z \cos Hz & +S \cos Z \sin Hz & +\sin Z \sin Hz \\ -S \sin Z \sin Hz & +S \cos Z \cos Hz & +\sin Z \cos Hz \\ 0 & -S \sin Z & +\cos Z \end{bmatrix} \tag{4}$$

2.2. The Uncertainty of the Convergences

Let us consider two object points (1 and 2), located in the same cross-section, whose relative cartesian coordinates in the instrument referential are:

$$X_2 = \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}, \quad X_1 = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} \tag{5}$$

The distance (D) between the object points can be computed, from its cartesian coordinates, by:

$$D = \sqrt{x_{12}^2 + y_{12}^2 + z_{12}^2}, \quad (x_{12} = x_2 - x_1, \text{etc.}) \tag{6}$$

The convergence of the object points between two epochs, t_1 e t_2, is:

$$\Delta D = D(t_2) - D(t_1) \tag{7}$$

Having in attention that the gradient of the distance is given by:

$$\nabla D = \begin{bmatrix} \frac{\partial D}{\partial x_{12}} & \frac{\partial D}{\partial y_{12}} & \frac{\partial D}{\partial z_{12}} \end{bmatrix} = \frac{1}{D} \begin{bmatrix} x_{12} & y_{12} & z_{12} \end{bmatrix} \tag{8}$$

the formula of the propagation of the variances allows the determination of the variance of the distance between points 1 and 2:

$$\mathrm{V}(D) = D^{-2} \; X_{12}^T \; (\Sigma_1 + \Sigma_2) \; X_{12} \tag{9}$$

where Σ_1 e Σ_2 are the matrices of variance-covariance of the vectors of the coordinates of the object points and where $X_{12} = X_2 - X_1$.

The uncertainty of the convergence, expressed by the standard deviation, is:

$$\sigma_{\Delta D} = \sqrt{V(D(t_2)) + V(D(t_1))} \tag{10}$$

If the variances $\mathrm{V}(D(t_1))$ and $\mathrm{V}(D(t_2))$ are identical then:

$$\sigma_{\Delta D} = \sqrt{2V(D)} \tag{11}$$

3. The Measurement Equipment

There is topographic equipment and also dedicated software that can be applied to continuous monitoring of tunnels, which usually demands high precision results. The software analyses the convergences in real time, in order to send out a signal, if the values exceed pre-defined limits. Table 1 presents the standard deviation of the values measured by a motorized tacheometer (Leica TCA2003) with *automatic target recognition* (ATR), obtained using prisms of high quality.

		Standard Deviation
Angular	automatic pointing	1 mm distances up to 200m
	manual pointing	0.15 mgon
Distance		1 mm + 1 ppm

Table 1: The standard deviation of a motorized tacheometer

In normal observation conditions and for short distances, the uncertainty of angular measurements, in automatic pointing mode, is constant (independent of the distance) and characterized by a linear standard deviation (1mm for the TCA2003).

But, under special conditions, the accuracy of the automatic target recognition can be improved [1]. These conditions, described to Leica TCA2003 and TCA1800 [1], comprise: short-range distances; constant light and dark background; prism clean, not fogged and oriented to the tacheometer; no atmospheric disturbances; the use of high precision prisms. Under these conditions, which are frequently verified at tunnels, accuracy can reach levels similar to manual pointing.

The recognition procedure of the ATR system is based upon CCD video technology and is not prism dependent, so several types of prisms can be used. In Figure 1 are presented some of the Leica prisms available.

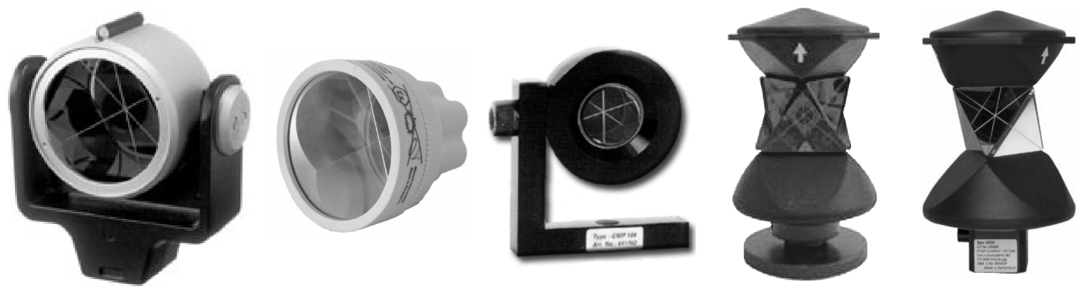

Figure 1: Leica prisms

The main characteristics of these prisms are:

1. Prisms GPH1P, GPR112 and GMP104 can be oriented to the tacheometer. Prisms GPH1P and GPR112 are the most precise ones; GMP104 is a mini prism. Using a TCA2003 and in average atmospheric conditions, the range of prisms GPH1P and GPR112 is 2500m while the range of mini prisms is 900m [2]. Prism GPR112, that is a version of prism GPH1P, was developed for underground work. Prism GPR112 and GMP104 are prepared to be screwed to surfaces.
2. The omni-direction prisms GRZ121 and GRZ4 don’t need to be oriented to the tacheometer. Each one consists of six individual prisms joined together. Using a TCA2003 and with average atmospheric conditions the range of prisms GRZ4 is 1500m [1]. The overall positioning accuracy of the prism GRZ4 is 5mm (in distance and in automatic pointing, [1]) but, when the pointing is made directly to one of the six prisms, an accuracy of better than 2mm can be achieved [3].

The previous prisms are not adequate for manual pointing since they don’t have any optical aiming mark. In Figure 2 are presented retro-reflecting targets that can be used for manual pointing.

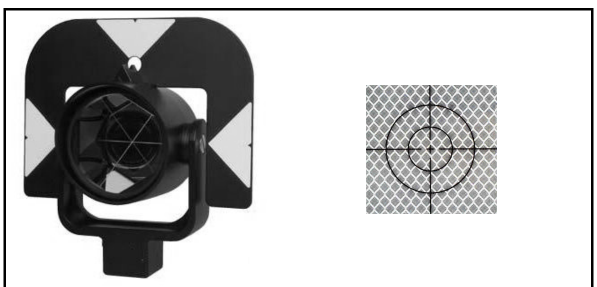

Figure 2: Leica retro-reflectors (manual pointing)

The main characteristics of these are:

1. GPR121 have similar characteristics to prisms GPH1P. They can be used both in automatic and manual pointing, this one due to the target plate.
2. Retro-reflecting targets have lower accuracies than glass prisms and can’t be used in automatic pointing. Their range is, usually, less than 200m. Distance accuracy is 3mm. Pointing accuracy depends much on the light conditions and on the distance tacheometer-target. At some distances, the reticle traces cover the crosshairs at the target, making the pointing quite inaccurate.

4. Example of Application

As example of application, let us consider a tunnel whose cross-section is a semicircle with a constant radius of 5m (Figures 3 and 4). In each cross-section are installed three object points (Figure 3), materialized by prisms, mounted on the tunnel lining, oriented to the tacheometer. One prism is installed on the crown (I). In the first cross-section a motorized tacheometer is permanently mounted on a bracket fixed on the wall.

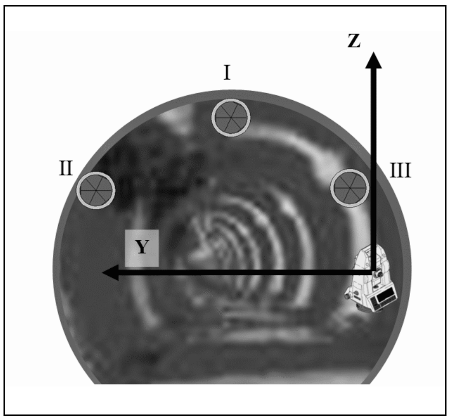

Figure 3: Position of the equipment in the cross-section

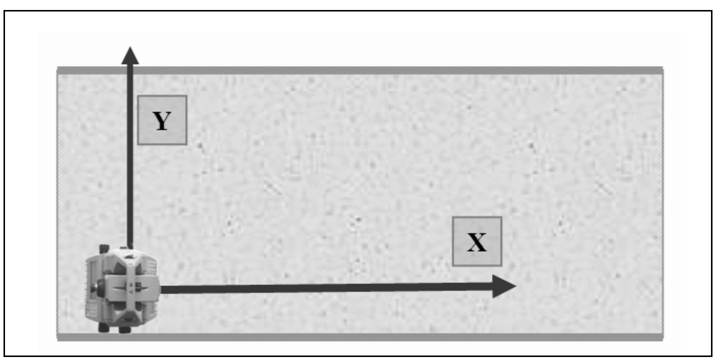

Figure 4: Axis orientations

The uncertainty of convergence will be computed taking into account four different scenarios, shortly described in Table 2.

Scenario	Description	Standard Deviation	
1	Automatic pointing to high precision prisms	pointing: 1mm	linear: 1mm
2	Manual pointing to high precision prisms	angular: 0.15mgon	linear: 1mm
3	Automatic pointing to high precision prisms. Indoor observations in stable atmosphere.	angular: 0.20mgon	linear: 0.1mm
4	Manual pointing to retro-tapes. Indoor observations in stable atmosphere.	angular: 0.60mgon	linear: 3mm

Table 2: Scenarios tested

Scenarios one and two will use values of standard deviations presented in Table 1. Scenarios three and four will use values of standard deviations estimated from measurements made by a tacheometer Leica TCA2003 in monitoring systems that include indoor observations, applying the procedures described in standard ISO 17123 [4].

In scenario three, it is assumed that the tacheometer is always the same and is permanently mounted. The special conditions inside tunnels allow the errors, which may occur during the measurement of distances [5], to be considered neglectable. For this reason, only random errors will have some effect on the distances.

Figure 5 presents the variation of the convergence measurement uncertainty of the chord II-III (Figure 3). It is supposed that there is a meteorological sensor at the tunnel so that distances are corrected from environmental effects.

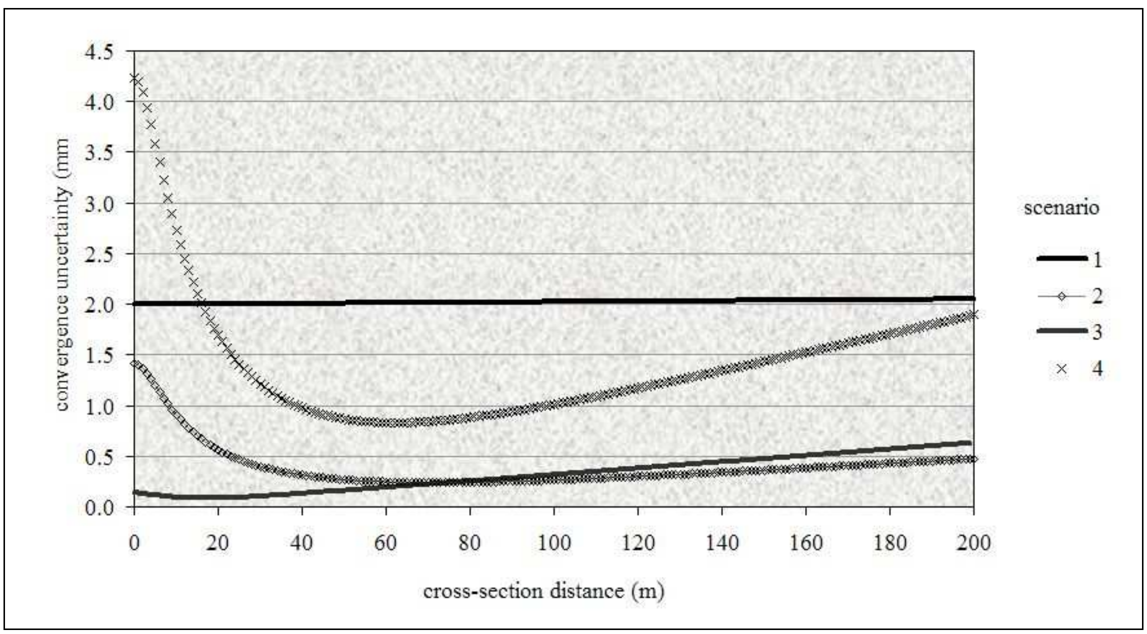

Figure 5: Uncertainty of convergence

The overall best results are obtained in scenario 3: the uncertainty varies from 0.1mm (cross-section at a distance of 17m away from the tacheometer) until 0.6mm (200m). In scenario 4 is interesting to notice that, for cross-sections at distances of 20m and beyond, the convergence measurement uncertainties are smaller that the ones of scenario 1. The convergence measurement uncertainties of chords I-II and I-III are not shown since they are similar to the convergence measurement uncertainties of chord II-III.

5. Conclusions

Motorized high precision tacheometers are an interesting option to the measurement of convergences in cross-sections of railroad and road tunnels. In favourable environmental conditions, which occur in the majority of tunnels, motorized tacheometers show a precision very similar to conventional tacheometry.

It is important to stress that good results are dependent on favourable observational conditions, with no turbulence, no wind, no thermal gradients, etc.. It has been reported, that abnormal results may occur, either from random errors in distance measurements or in ATR pointings, mainly as a result of perturbations in the atmosphere. So, when abnormal results are obtained, new observations should be carried out to confirm the previous measurements.

References:

[1] LEICA: TPS - System 1000 – Electronic Theodolites and Total Stations, Leica Geosystems AG, Heerbrugg, 2000.

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[4] ISO: International Standard ISO 17123. Optics and Optical Instruments - Field Procedures for Testing Geodetic and Surveying Instruments, International Organization for Standardization, Geneva, 2001.

[5] Rüeger,J.M.: Electronic Distance Measurement – An Introduction, 4th Edition, Springer Verlag, Berlin/Heidelberg/New York, 1996.