A NEW GPS BASELINE PROCESSING ALGORITHM FOR ENGINEERING SURVEYS

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Abstract: A detailed algorithm for GPS baseline processing using phase observations is presented in this paper. The algorithm was the basis to elaborate a new software which was created mainly for research purposes, especially for testing different numerical methods on each stage of the GPS baseline determination. The baselines analysed according to the described algorithm are expected to form a data file, which may by utilized in engineering surveys.

1. Introduction

The automated, monitoring deformation systems necessitate implementation of real time mode baseline processing algorithm. This algorithm must ensure fast calculations and oportunity of instant detecting the changes of point position. Sequential adjustment algorithm has these features. The way of using the sequential adjustment algorithm to baseline processing in real time mode is described below.

2. Observation equations

In proposed algorithm two types of observation are used: the pseudoranges and carrier phases. We can show the observation equations for both types of observations as follow[1],[2],[3],[4]:

$$\Phi=\tfrac{f}{c}\rho+f(dt^s-dt_r)+\tfrac{f}{c}\delta^{trop}-\tfrac{f}{c}\delta^{jon}+N+\varepsilon_\Phi \qquad (1)$$

$$P=\rho+c(dt^s-dt_r)+\delta^{trop}+\delta^{jon}+\varepsilon_p$$

where:

Φ - measured carrier phase

f - carrier frequency

ρ - geometric range receiver-satellite

c - vacuum speed of light

dt^s - offset of satellite clock

dt_r - offset of receiver clock

δ^{trop}, δ^{jon} -delays due to the troposphere and ionosphere

ε_{Φ}, ε_p - the effect of measurement noise for carrier phases and pseudoranges respectively

In proposed algorithm double differenced carrier phase observations are used:

$$\nabla\Delta\Phi=\nabla\Delta\rho+\nabla\Delta N+\nabla\Delta\varepsilon \tag{2}$$

It is assumed that the clock offsets and the effect of ionosphere are removed by double differencing the observations.

3. Ambiguity resolution

To resolve ambiguities both types of observation: pseudoranges and carrier phases are used.

Therefor a preliminary adjustment is performed. In this adjustment following functional model is used:

$$\underline{V}=\underline{A}\underline{X}+\underline{L}\ , \tag{3}$$

where:

$\underline{V}$ - corrections vector

$\underline{A}=\begin{bmatrix} A_P \\ A_{DD} \end{bmatrix}$ A_p, A_{DD} - functional model matrices for pseudoranges and double differenced carrier phases respectively

$\underline{X}=\begin{bmatrix} X_c \\ X_a \end{bmatrix}$ X_c-coordinates increment vector, X_a - ambiguity vector

$\underline{L}=\begin{bmatrix} L_p \\ L_{DD} \end{bmatrix}$ L_p, L_{DD} - free terms vectors for pseudoranges and double differenced carrier phases respectively

Statistical model can be written as:

$$\underline{C}=\delta^2\underline{Q} \tag{4}$$

where:

$\underline{C}$ - covariance matrix

$\delta^2=\dfrac{V^TC^{-1}V}{n-m}$ - variance coefficient

(n - number of observations, m- number of parameters

$\underline{Q}=\begin{bmatrix} Q_p & \\ & Q_{DD} \end{bmatrix}$ Q_p, Q_{DD} -cofactor matrices for pseudoranges and double differenced carrier phases respectively

Hence the solution of least squares estimation is the following vector:

$$\underline{X}=(\underline{A}^T\underline{C}^{-1}\underline{A})^{-1}\underline{A}^T\underline{C}^{-1}\underline{L} \tag{5}$$

and his variance matrix:

$$C_X=\delta^2\,(A^TC^{-1}A)^{-1} \tag{6}$$

Matrix C_X has following structure:

$C_X = \begin{bmatrix} C_c & \\ & C_a \end{bmatrix}$, C_c, C_a - covariance matrices of coordinates and ambiguites respectively.

Algorithm of preliminary adjustment can be shown as follow:

1. Acquisition of observation data (pseudorange and phases) from the first epoch
2. Adjustment
3. Testing the following condition:

   $$\text{maximum}(\text{diag}(C_a)) < \delta^2_{max} \quad (7)$$

   where δ^2_{max} is constant
4. If condition (7) returns false then number of observations is increased by adding observation set from next epoch in next adjustment.
5. When condition (7) returns true the preliminary adjustment is finished. Integer ambiguities are calculeted from last step using vector X_a and C_a matrix with LAMBDA method [5].
6. Finally adjustment of the double differenced carrier phases with fixed, integer ambiguites is performed. The functional model of this adjustment can be presented by means of the following system of the correction equations:

   $$\underline{V}_{DD}=\underline{A}_{DDc}\underline{X}_c+\underline{L}_{DDc} \quad (8)$$

where:

$\underline{L}_{DDc}=\underline{L}_{DD}+\underline{A}_{DDa}\underline{X}_{afix}$

$\underline{A}_{DDc}$, $\underline{A}_{DDa}$ - submatrices of A_{DD} refering to coordinates and ambiguites respectively

$\underline{X}_{afix}$ - vector of fixed, integer ambiguites

4. Sequential adjustment

Functional model for the sequential adjustment reads as follow:

$$\underline{V}_s=\underline{A}_s\underline{X}_c+\underline{L}_s \quad (9)$$

where:

$\underline{V}_s$ - residuals vector

$\underline{A}_s = \begin{bmatrix} E \\ A_{DDc} \end{bmatrix}$ E is 3x3 dimension unit matrix

$\underline{L}_s = \begin{bmatrix} 0 \\ L_{DDc} \end{bmatrix}$ 0 is 3x1 dimension vector of zeros

Statistical model can be presented in the form of following covariance matrix:

$$\underline{C}_s = \delta^2\underline{Q}_s \quad (10)$$

where:

$$\underline{Q} = \begin{bmatrix} Q_c & \\ & Q_{DD} \end{bmatrix}, \; Q_c \text{ - cofactor matrix for coordinates}$$

If preliminary adjustment observations set consists of observations taken from n epochs, then sequential adjustment starts with n+1 epoch (the n+1 epoch of baseline processing is the first epoch of sequential adjustment). In each successive epoch separate adjustment is performed. The matrix $\underline{A}_s$ and the vector $\underline{L}_s$ are formed on the basis of coordinates obtained from previous adjustment and carrier phases from present epoch. In the first epoch of sequential adjustment elements of matrix A_{DDc} and vector L_{DDc} are determined on the basis of coordinates taken from preliminary adjustment and fixed ambiguities. Before matrix $\underline{A}_s$ and vector $\underline{L}_s$ are formed, cycle slips must be detected. Solution of this problem is e based on triple differenced carrier phases analysis. Triple differences are formed as differences of double differenced carrier phases from last three successive epochs. This values (for each pair of satellites) are stored and updated in two-elements vectors. It is assumed that cycle slip appears if rounded difference of that two elements differs from zero. If cycle slip is detected the appropriate value of ambiguity is changed.

5. Results of the test

The algorithm was applied to raw GPS data. The results of preliminary adjustments are given in Fig. 1. Mean errors of ambiguities were calculated as square roots of diagonal elements $\underline{C}_a$ from formula (6). In the test it took 36 epochs (with interval=20 sec.) of observations before the mean errors of ambiguities were lower than δ_{max}=1.5.

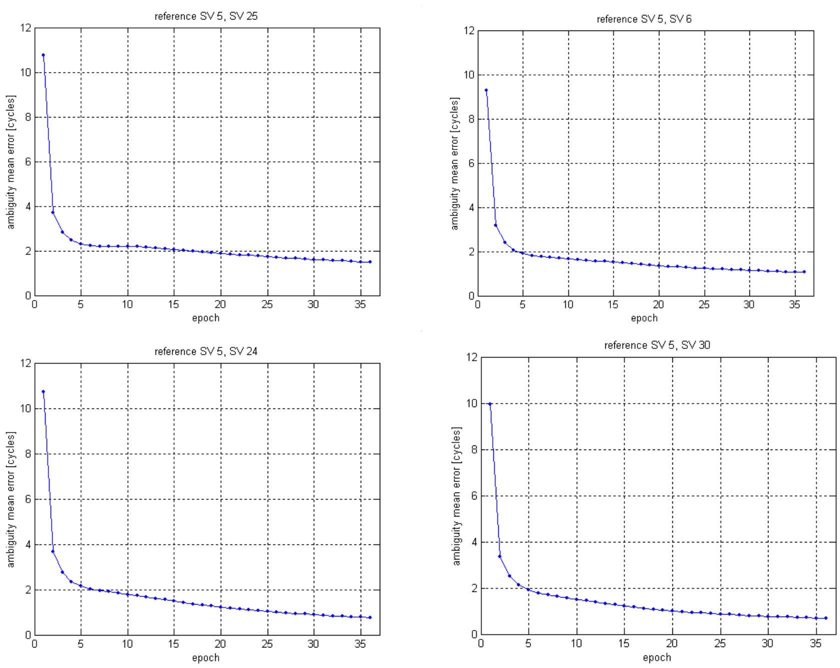

Fig. 1 Mean errors of ambiguities

In Fig. 2 differences of parameter values in succesive epochs and their mean errors are schown.

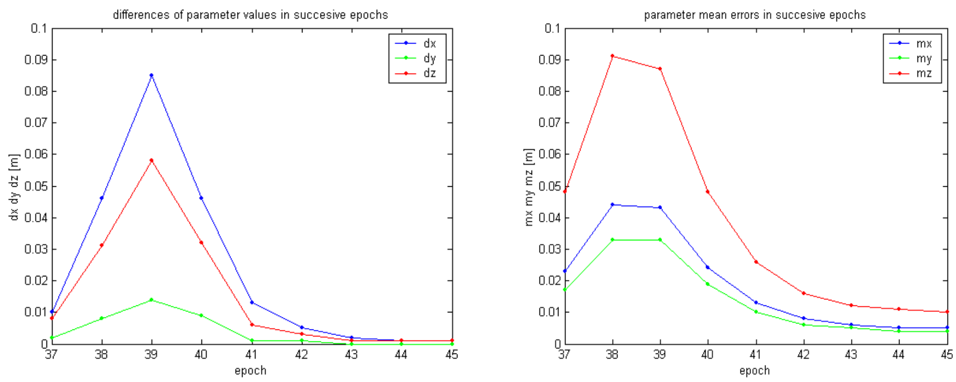

Fig. 2 Differences of parameter values in successive epochs and their mean errors

After about six epochs from start of sequential adjustment coordinates stopped varying.

6. Final remarks

Algorithm described in this paper offers the possibility of detecting instant change in point position. It is proper for deformation measurements purposes.

References:

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