GEOPOTENTIAL NUMBERS

- Because, when we do levelling, we measure differences in height "dH" measured in a linear (metric) world, we are not determining the ***true*** difference in gravitational potential - "dW" - between two points.
- To find "dW" we should scale the observed dH by the gravitational acceleration **g**, invoking the expression in Eq (1), that is
  - **g** = dW/dH ...(1)

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  - **g** = dW/dH ...(1)

- If we take our levels from a reference surface (think height datum) whose geopotential is stated (eg, W_0), then the geopotential of our levelled station P is
  - $W_P = W_0 + dW_{0P}$, or
  - $W_P = W_0 + \mathbf{g}.dH_{0P}$

GEOPOTENTIAL NUMBERS

By convention, we call the value ($W_P – W_0$) the Geopotential Number, or

$$C_P = -(W_P - W_0) = -g.dH_P$$

The negative is to reflect the fact that an increase in height invokes a decrease in potential.

- NOTE: Over short distances, or in regions of low gravitational variation, this difference will be insignificant.

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The units for geopotential numbers are in "$10m^2s^{-2}$" an unfriendly unit called kilogal metres (could just as easily be called Nics or Kevs).

This ensures the numerical value of C_P is close to the value of H_P (in metres), or

$$C_P = 0.98H_P \text{ (approx)}$$

(See Torge, Geodesy, p.45)

THE GLOBAL GEOID & GEOPOTENTIAL NUMBERS

The Geopotential Number, or

$$C_P = -(W_P - W_0) = - g. dH_P$$

If we could somehow measure W_P and define W_0, we would have the ideal height system

a) A well-determined and unambiguous datum (W_0), and

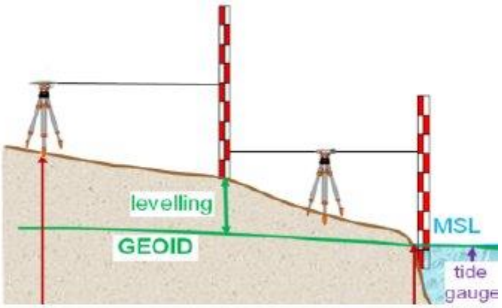
b) The point's true difference in elevation related to this datum

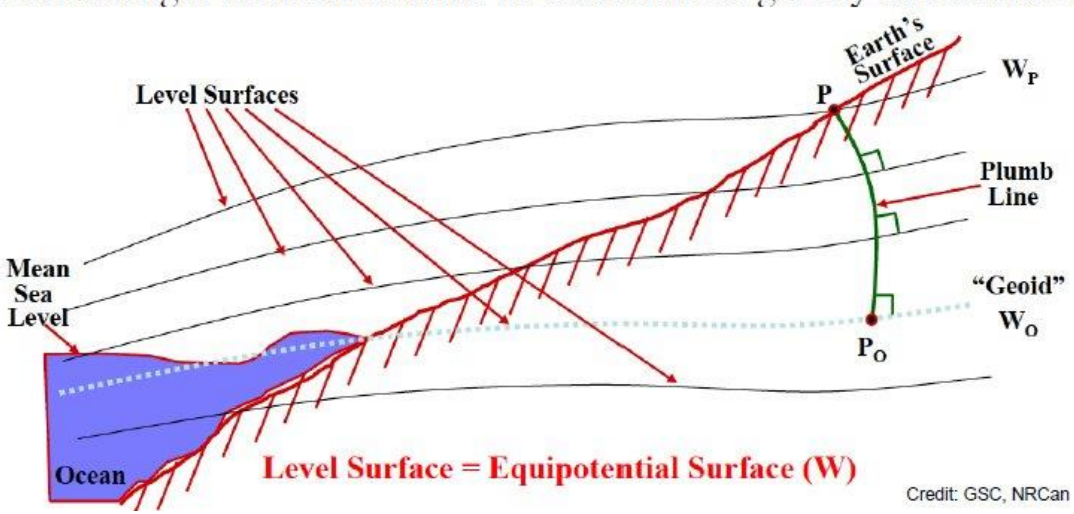
GEOPOTENTIAL NUMBERS

- **Definition of orthometric heights**

$$H_P = \frac{C_P}{\overline{g}} = \frac{W_o - W_P}{\overline{g}}$$

  - C … *geopotential number*
  - H … length of the plumb line from P to the geoid
    - Requires the average gravity value, $\overline{g}$, along the same path
    - Levelled height differences must be corrected for gravity to become ΔH





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a) A well-determined and unambiguous datum (W_0), and

b) The point's true difference in elevation related to this datum

But, the big question is - How do we determine W_0

THE GLOBAL GEOID

In the following slides I quote verbatim from the report by Prof. Laura Sanchez and the IAG Working Group on the best estimate for W_0

A new best estimate for the conventional value W_0

- Final Report of the WG on Vertical Datum Standardization -

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Department of Mathematics and Descriptive Geometry, Faculty of Civil Engineering, Slovak University of Technology in Bratislava, Slovakia
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Geographic Service of the Czech Armed Forces, Czech Republic

26th IUGG GENERAL ASSEMBLY 2015
PRAGUE, CZECH REPUBLIC
PRAGUE CONGRESS CENTRE
JUNE 22–JULY 2, 2015
Earth and Environmental Sciences for Future Generations

Introduction

- W_0 is understood as the potential value of the geoid;
- Since there are an infinite number of equipotential surfaces, the geoid is to be defined arbitrarily by convention;
- Usual convention: the geoid is the equipotential surface of the Earth's gravity field that best fits (in a least square sense) the undisturbed mean sea level;
- Since to satisfy this condition is not possible and since the sea level changes, a convention about mean sea level (time span and area) is also needed:
  - – mean value at a local tide gauge $W_0 = W_0^{(i)}$
  - – mean value a several tide gauges $W_0 = \frac{1}{n}\sum_{i=1}^{n} W_0^{(i)}$
  - – potential value of a best fitting ellipsoid in ocean areas $W_0 = U_0$
  - – mean value over ocean areas sampled globally $\int_S (W - W_0)^2 \, dS = \min$

W_0 and the IERS Conventions

A new best estimate for the conventional value W_0
IUGG 2015, 2015-06-27

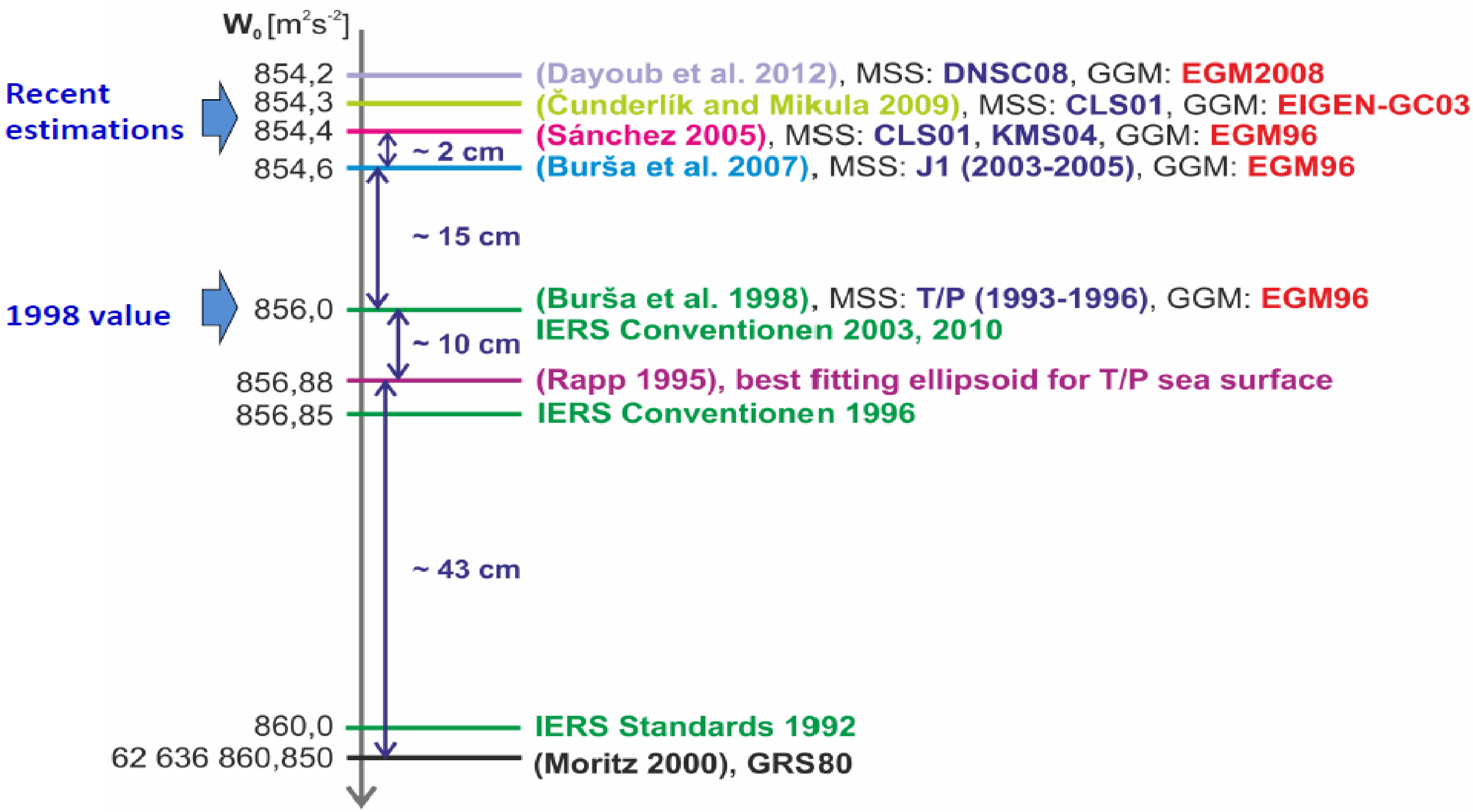
- In 1991, the International Astronomical Union introduced timescales for the relativistic definition of the celestial space-time reference frame;
- The relationship between Geocentric Coordinate Time (TCG), and Terrestrial Time (TT) depends on the constant $L_G = W_0 / c^2$
- For this reason, the IERS Conventions included a W_0 value and updated this value regularly according to new best-estimates:

Year	W_0	L_G
1991	62 636 860 ± 30 m^2s^{-2} (Chovitz 1988)	6.969 291 × 10^{-10} ± 3 × 10^{-16} (IAU 1991, Recommendation IV, note 6)
1992	62 636 856.5 ± 3 m^2s^{-2} (Burša et al. 1992)	6.969 290 19 × 10^{-10} ± 3 × 10^{-17} (Fukushima 1995)
1995	62 636 856.85 ± 1 m^2s^{-2} (Burša 1995a)	6.969 2903 × 10^{-10} ± 1 × 10^{-17} (McCarthy 1996, Tab. 4.1)
1999	62 636 856.0 ± 0.5 m^2s^{-2} (Burša et al. 1998, Groten 1999)	6.969 290 134 × 10^{-10} (as defining constant) (IAU2000, Resolution B1.9)

- In 2000, L_G is declared as "defining constant", i.e. it should not change with new estimations of W_0. The corresponding W_0 value is the best-estimate available in 1998.

3

Recent W_0 computations (since 2005) based on newer models of the sea surface and the Earth's gravity field



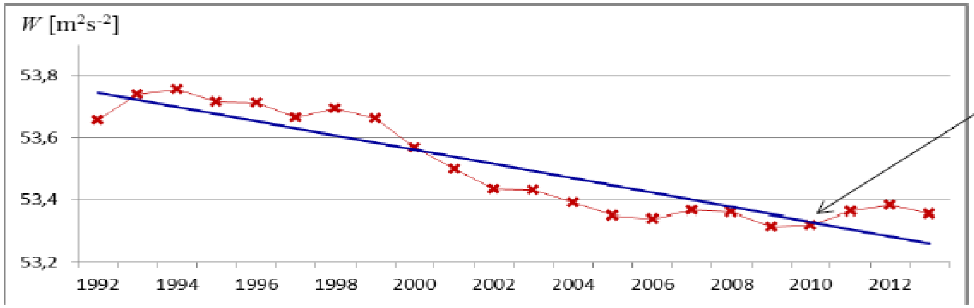
Conclusions

A new best estimate for the conventional value W_0
IUGG 2015, 2015-06-27

1) Computations carried out within the WG-VDS demonstrate that the 1998 W_0 value (62 636 856.0 ± 0.5 m^2s^{-2}) is not in agreement (and consequently it is not reproducible) with the newest geodetic models describing geometry and physics of the Earth.
2) The 1998 W_0 value is not suitable as a conventional reference value and a *better estimate* for W_0 has to be adopted by the IAG for the definition and realization of the IHRS.
3) As reference level, the conventional value W_0 has to be fixed (without time variations); but it has to have a clear relationship with the sea surface (as convention for the realization of the geoid).

Conclusions

4) We propose to adopt the potential value obtained for the year 2010 after fitting the yearly W_0 estimations by means of a lineal regression:



W_0 = 62 636 853.353 m^{2}s^{-2}
rounded to
W_0 = 62 636 853.4 m^{2}s^{-2}

5) The formal error of this value is **± 0.02 m^{2}s^{-2}**. However, as convention the adopted W_0 is understood free of error.

6) The introduction of a reference W_0 value is not accepted by the whole geodetic community. There are a variety of approaches to avoid a W_0 value.

7) Results provided by the WG-VDS are for those approaches requesting a reliable W_0 value.

THE GLOBAL GEOID

Grateful thanks to Prof. Laura Sanchez and the IAG Working Group for this exacting work on the best estimate for W_0

As noted, the current best estimate for this constant is

62 636 853.4 m^2s^{-2} +/- 0.02 m^2s^{-2}

Compared with the 1998 value of

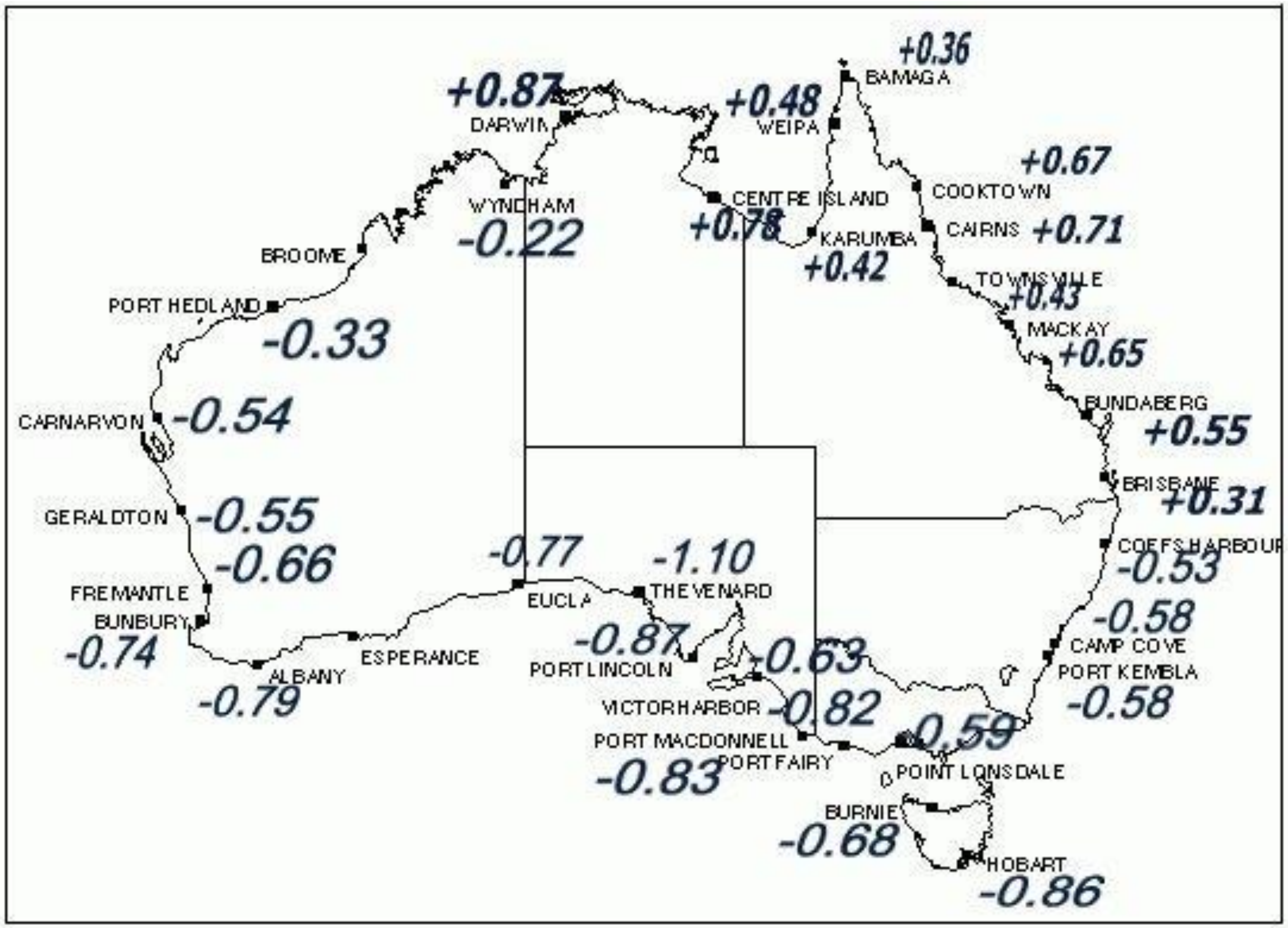
62 636 856.0 m^2s^{-2} +/- 0.5 m^2s^{-2}

This latest version is smaller by about **3 m^2s^{-2}**

and of course, benefits from the increase in both the quantity and quality of the altimetry data over the oceans, and the improvement in the global gravity models, and the models for e.g. the mean sea surface

AHD Tide Gauges with respect to a Global Geoid,

W_0 = 62 636 856.0 m^2s^{-2} (Kearsley, 2007 , IUGG Perugia)

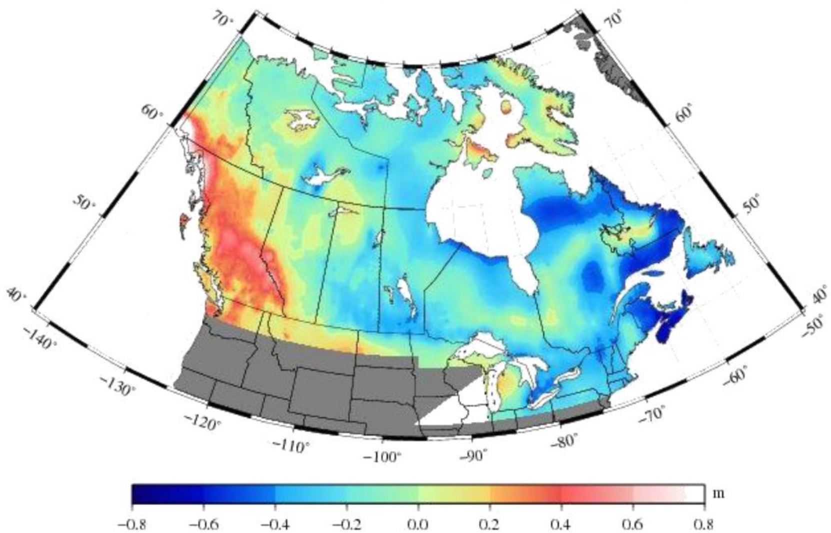


Sponsors: Land Information New Zealand Toitū te whenua, Leica Geosystems, Trimble

Thus end my Reflections - Thank you

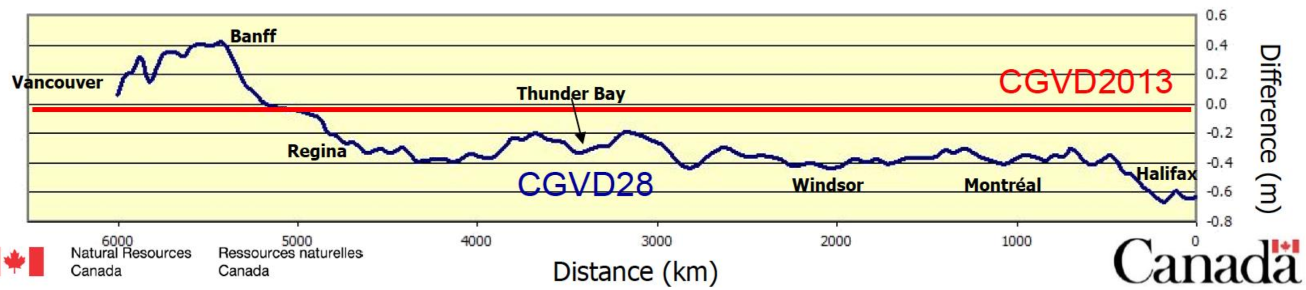
Distortions in the Canadian Vertical Datum with respect to the Geoid

CGVD28(HTv2.0) – CGVD2013(CGG2010)



Preliminary values
$H_{CGVD2013} - H_{CGVD28}$

St John's	-37 cm
Halifax	-64 cm
Charlottetown	-32 cm
Fredericton	-54 cm
Montréal	-36 cm
Toronto	-42 cm
Winnipeg	-37 cm
Regina	-38 cm
Edmonton	-04 cm
Banff	+55 cm
Vancouver	+15 cm
Whitehorse	+34 cm
Yellowknife	-26 cm
Tuktoyaktuk	-32 cm



Natural Resources Canada
Ressources naturelles Canada
Canada

Sponsors: Land Information New Zealand Toitū te whenua; Leica Geosystems; Trimble.

SECTION 5
GNSS/GPS HEIGHTING

For many engineering applications, especially those requiring drainage or transfer of water and other fluids, the parameter "h" or "Δh“, derived from GNSS, is not directly useable.

Applications of “h” - GPS height

For many engineering applications, especially those requiring drainage or transfer of water and other fluids, the parameter "h" or "Δh" is not useable.

However, “h” may be useful in cases where *changes* in height are being studied (in which case it does not matter which height system is invoked!).

E. g.,

- Monitoring vertical motions of plate tectonics
- Monitoring subsidence of city areas and buildings.
- Predict the dangerous conditions, such as subsidence caused by mining for extraction of coal, oil, etc....

"Orthometric" Height from GNSS/GPS Height

On the other hand, for most engineering applications, we need to transform the ellipsoidal height "h" (or "Δh") derived from our GNSS/GPS measurement to the orthometric height "H" (or "ΔH") for use in surveying and civil engineering design.

"Orthometric" Height from GNSS/GPS Height

To a close approximation;

h = H + N or

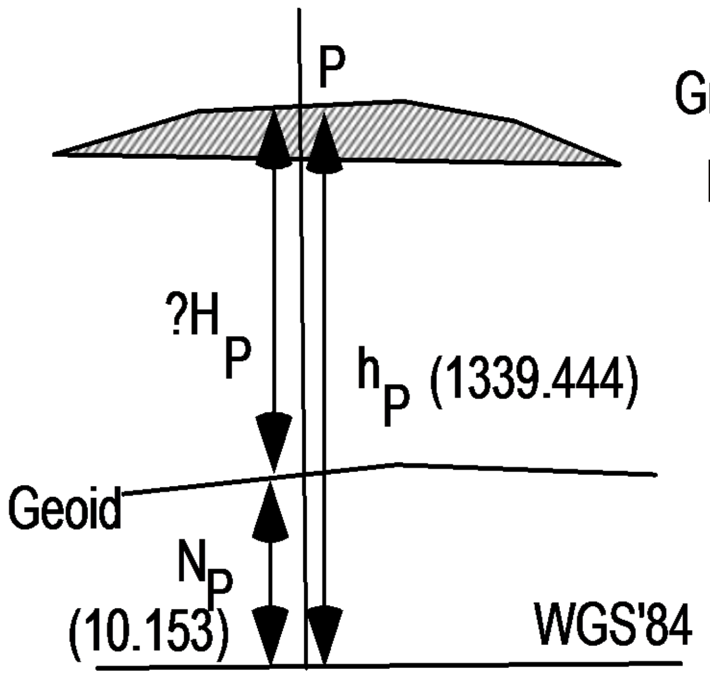
H = h – N

where N is the geoid height

Eg., if "h_P" from GPS with respect to WGS'84 is 1339.444, and the height of the geoid, "N_P", above the WGS'84 ellipsoid was 10.153 m, then

$\boldsymbol{H_P}$ *= 1339.444 - 10.153*

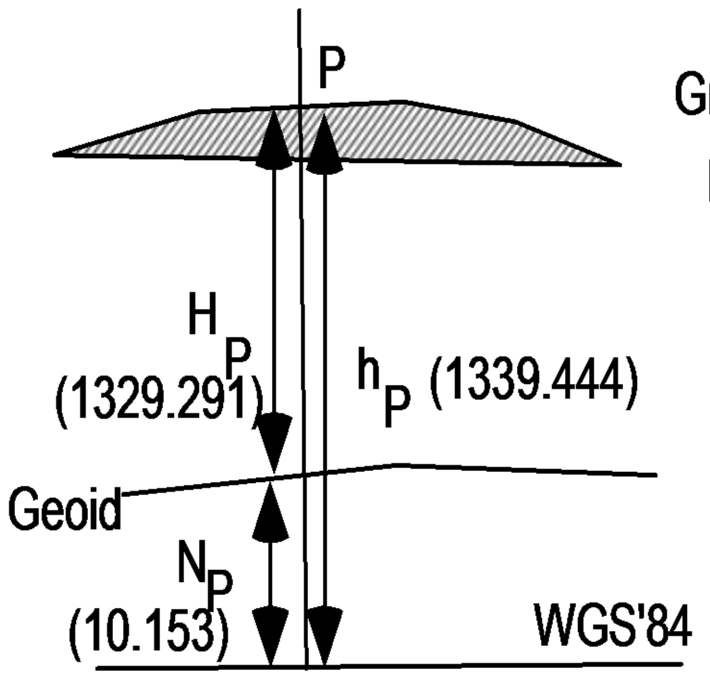
= ***1329.291*** *m.*



"Orthometric" Height from GNSS/GPS Height

This approach is practical when we have a good (?sufficiently accurate) knowledge of the geoid height, for the purpose of the survey.

E.g., if we only need to height to +/- 20cm, and our geoid heights are known to be accurate to that level.



Orthometric Height from GNSS/GPS Height

COMMENT:

The datum for the point positioning becomes significant in this case, as it is vital that the geoid relates (or is transformable) to the Height datum for the height you are trying to establish.

A number of National Geodetic Authorities are now producing a "Hybrid"-geoid which is a gravimetric geoid distorted to fit the national height datum!

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For example, in Australia there is a hybrid geoid model – AUSGeoid09 – which is essentially the gravimetric geoid fitted to the AHD; this enables heights from GNSS to be transformed directly to the AHD. The earlier versions of AUSGeoid (AUSGeoid93/98) were purely gravimetric solutions so provided corrections from the ellipsoid to the geoid.

see http://www.ga.gov.au/ausgeoid/nvalcomp.jsp.

Orthometric Height from GNSS/GPS Height

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Likewise the US Geodetic Authority has been leading the community in developing a series of side-by-side gravimetric-only and hybrid geoid models. The former has vlaue for the scientific community, whilst the latter is of great practical use to the surveying practitioner. They are aiming at a geoid surface

See http://vdatum.noaa.gov/

They also provide the user with the expected uncertainties

http://vdatum.noaa.gov/docs/est_uncertainties.html

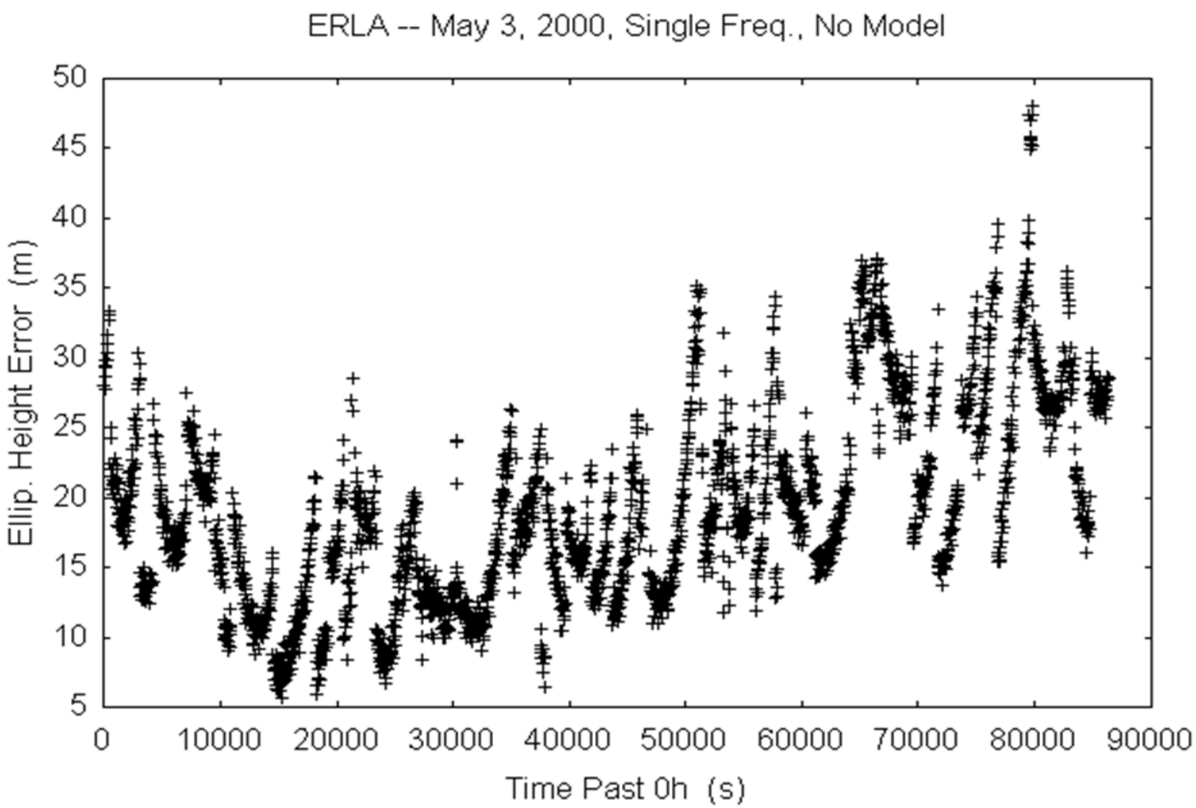
Orthometric Height from GNSS/GPS Height

Systematic errors or biases in “h”

(a) Satellite dependent

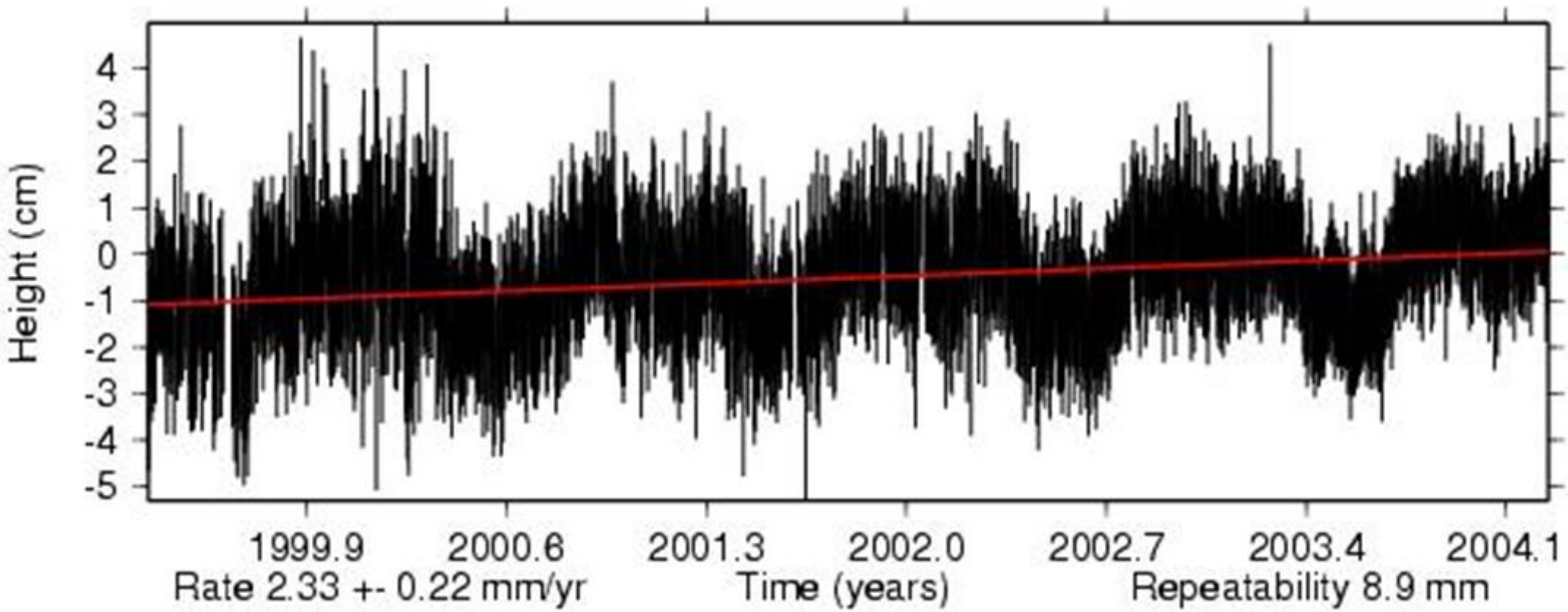
- * Orbit errors because of an incorrect satellite ephemeris
- * Satellite clock model biases
- (b) Station dependent
- * Receiver clock biases
- * Station coordinates
- (c) Observation dependent
- * Ionospheric delay
- * Tropospheric delay
- * Carrier phase ambiguity
- * Antenna height mismeasurement

Ionospheric effects on GPS height



- See: http://www.ngs.noaa.gov/FGCS/info/sans_SA/iono-ht

Seasonal variation of absolute GPS height



Height precision from GNSS/GPS Height

GPS Method	Observables	Horizontal (m)	Vertical (m)
ABSOLUTE	**POSITIONING**		
Single point	C/A	2 - 5	5 - 10
RELATIVE	**POSITIONING**		
Static DGPS	C/A	0.5 - 2.0	1.0 -3.0
Static carrier phase	L1	0.02	0.03
ditto	L1 & L2	0.005	0.02
Rapid-static	L1 & L2	0.02	0.03
Non-static	**Post-processed**		
DGPS	C/A	2 - 5	3 - 8
Continuous kinematic	L1	0.03	0.05
ditto	L1 & L2	0.01	0.02
Stop/Go kinematic	L1 & L2	0.01	0.02
Non-static	**Real Time**		
DGPS	C/A	3 - 5	4 - 8
Continuous kinematic	L1	0.1	0.2
ditto	L1 &L2	0.05	0.1
Stop/Go kinematic	L1	0.03	0.05
ditto	L1 &L2	0.02	0.03

Orthometric Height from GNSS/GPS Height

- We note that we can determine Δh across a line with GPS more precisely than we could find "h" at each of the terminals of that same line. For similar reasons, we can find "ΔN" across a line by gravimetric solutions more precisely than we determine "N" at any one point.

(Why? Because the systematic errors in the h, N values observed/computed at the line terminals are significantly the same, and will largely cancel in the differences).

Orthometric Height from GNSS/GPS Height

We note that we can determine Δh across a line with GPS more precisely than we could find "h" at each of the terminals of that same line. Similarly we can find "ΔN" across a line by gravimetric solutions more precisely than we determine "N" at any one point. (Why? Because the systematic errors in the values observed/computed are held in common, and will largely cancel in the differences).

For this reason, we obtain the highest precision in GPS heighting when we operate in this "relative" mode. In other words, we establish the difference in orthometric height across the line by applying the difference in geoid height to the difference in ellipsoidal height across the line.

Height precision from GNSS/GPS Height

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ABSOLUTE	**POSITIONING**		
Single point	C/A	2 - 5	5 - 10
RELATIVE	**POSITIONING**		
Static DGPS	C/A	0.5 - 2.0	1.0 -3.0
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Non-static	**Post-processed**		
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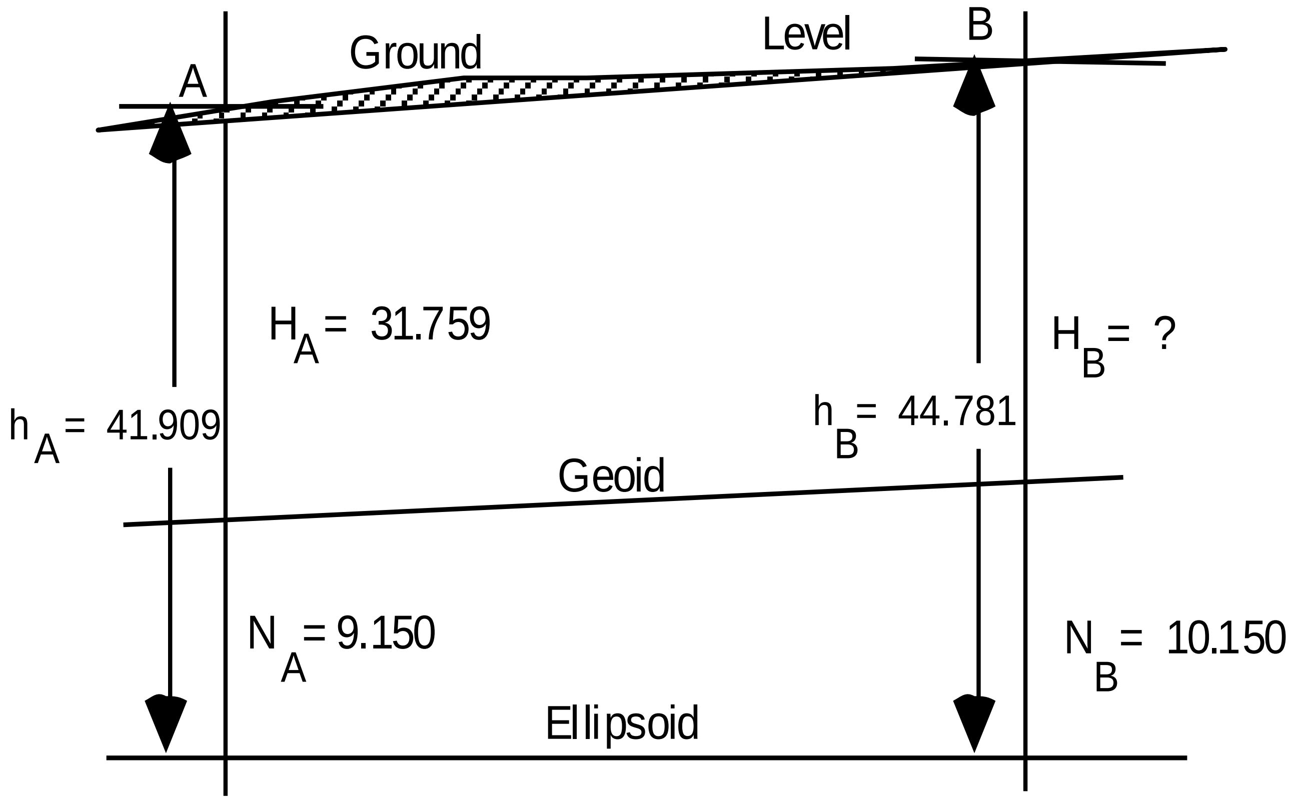
Height difference transformation - "Δh" to "ΔH

Defining

$\Delta H_{AB} = H_B - H_A$

$\Delta h_{AB} = h_B - h_A$, and

$\Delta N_{AB} = N_B - N_A$



Height difference transformation - "Δh" to "ΔH"

We can say

$H_B = H_A + \Delta H_{AB}$, or

$H_B = H_A + (H_B - H_A)$

By definition, this will equal

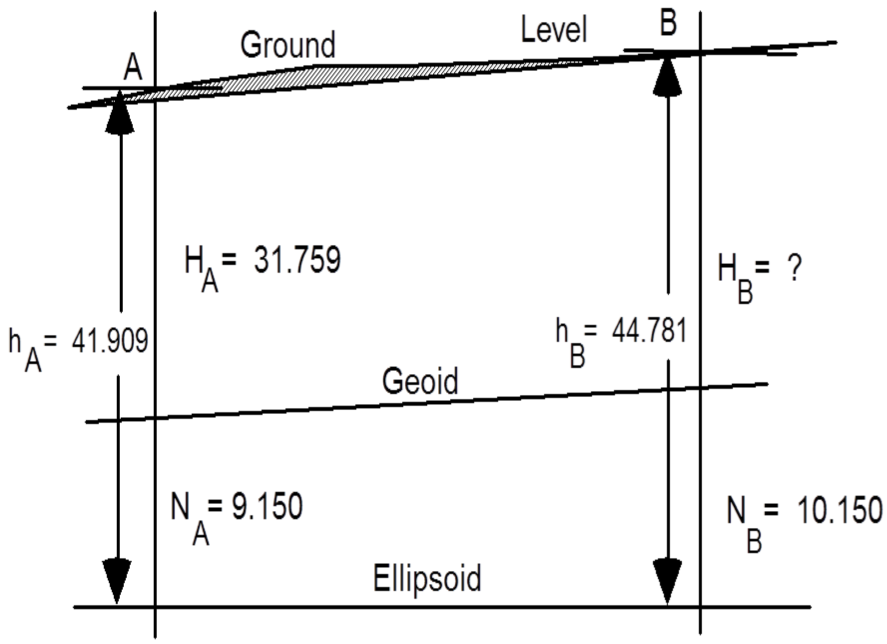
$H_B = H_A + (h_B - N_B) - (h_A - N_A)$

Rearranging terms

$H_B = H_A + (h_B - h_A) - (N_B - N_A)$

or

$\mathbf{H_B = H_A + \Delta h_{AB} - \Delta N_{AB}}$



Height difference transformation - "Δh" to "ΔH"

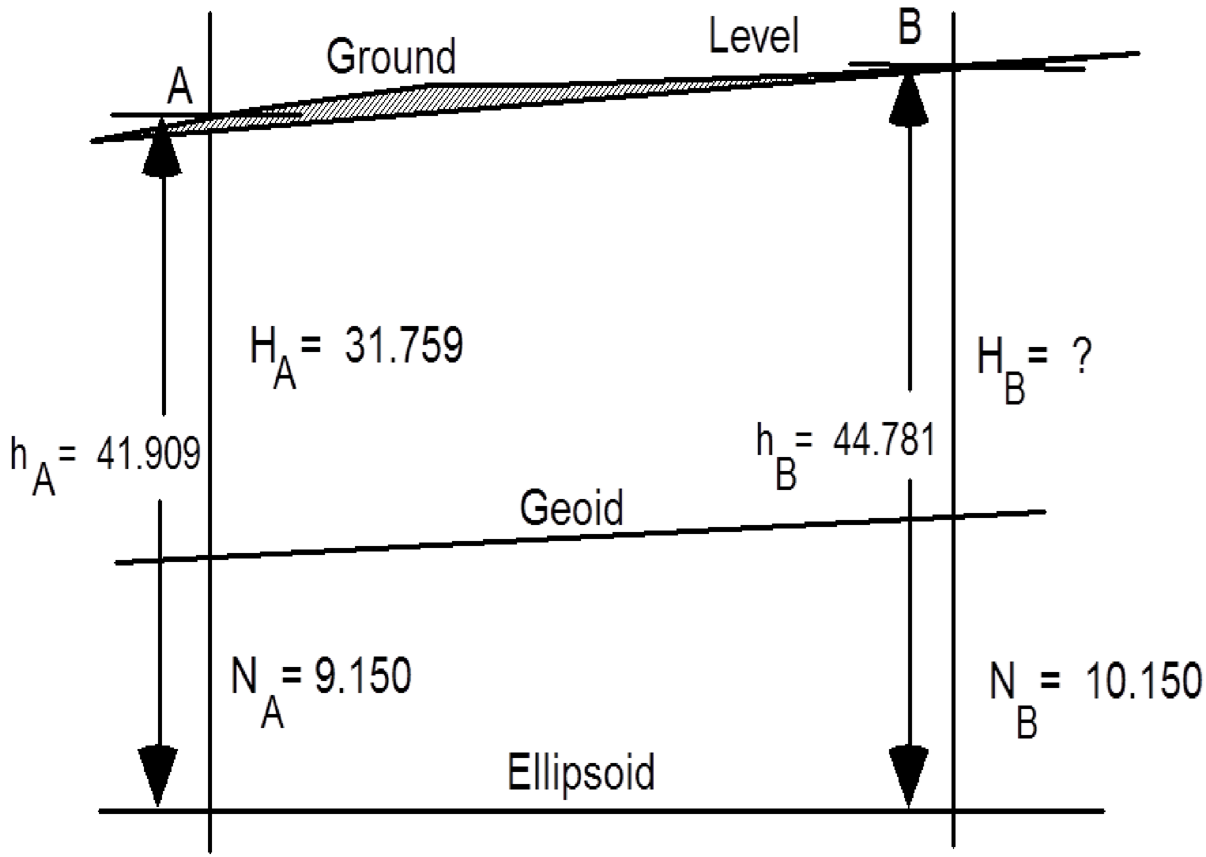
We can say

$H_B = H_A + \Delta h_{AB} - \Delta N_{AB}$

THUS

$= 31.759 + 2.872 - 1.000$

Or **HB = 33.631 m.**



The Expected Error in H from GPS-Gravimetry

We make the reasonable assumption that there is no correlation between ΔN_{AB} and Δh_{AB}, and find

$$\sigma^2 H_B = \sigma^2 H_A + \sigma^2 \Delta h_{AB} + \sigma^2 \Delta N_{AB}$$

Thus, if we want to find the error in the computed value of H_B relative to H_A, we assume that H_A is error-free or fixed, ie that $\sigma^2 H_A$ is effectively zero.

Example: Provided that H_A is error-free or fixed, i.e. that $\sigma^2 H_A$ is zero, if Δh from GPS can be 0.05 m, and ΔN from gravimetry is 0.1m, we find

$$\sigma^2 H_B = 0 + 25\ \text{cm}^2 + 100\ \text{cm}^2 = 125\ \text{cm}^2$$

namely $\sigma H_B \cong 11$ cm

The Expected Error in H from GPS-Gravimetry

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Thus, if we want to find the error in the computed value of H_B relative to H_A, we assume that H_A is error-free or fixed, ie that $\sigma^2 H_A$ is effectively zero.

Orthometric Height from GNSS/GPS Height

Comment:

Another very important reason for using the relative mode is that the problem relating to datums (GNSS ***AND*** Geoid) is significantly overcome, because this information is embedded in the values of the base station to which you are relating your height (see comments above on point positioning).

Orthometric Height from GNSS/GPS Height

Main Message

We are entering a new era of height determination, where heights measured by GNSS, combined with precise geoids, which result from detailed terrestrial, airborne and orbit studies of the Erath's gravity field, enable us to recover the Orthometric Heights of points to the 3rd order precision or better – certainly in flat to undulating terrain.

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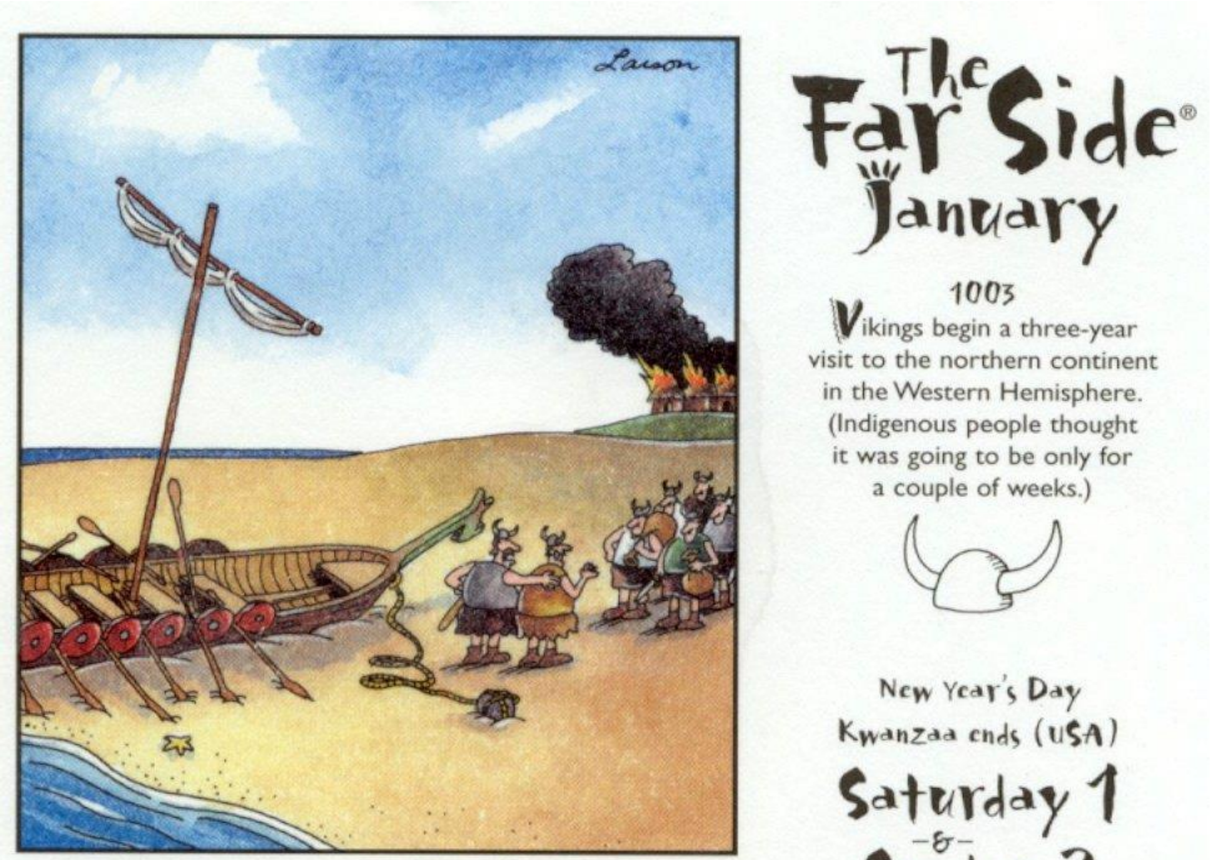
These heights can be related to a Global Geoid – of well-defined W0 – so that the Global Positioning systems employed can be used both in the local and regional context.

Vertical Reference Frames in Practice

THANK YOU

(But beware of the tides!!)



"Everyone can just put down their loot and plunder, and Sven here—yes, old Sven, who was in charge of reading the tide chart—has something to say to us all."