Crustal Deformation Modeling Theory and Examples

Manabu Hashimoto, DPRI, Kyoto University

Contents

- Theory of measurements of displacements
- Interpretation of measured displacements
- Recent examples

How to Measure Displacements?

- Comparison of coordinate before and after the event
  - GNSS, VLBI, SLR, Triangulation, Trilateration, Leveling
- Comparison of distance before and after the event
  - SAR interferogram

Displacement and Strain

- Displacement: Difference of coordinates

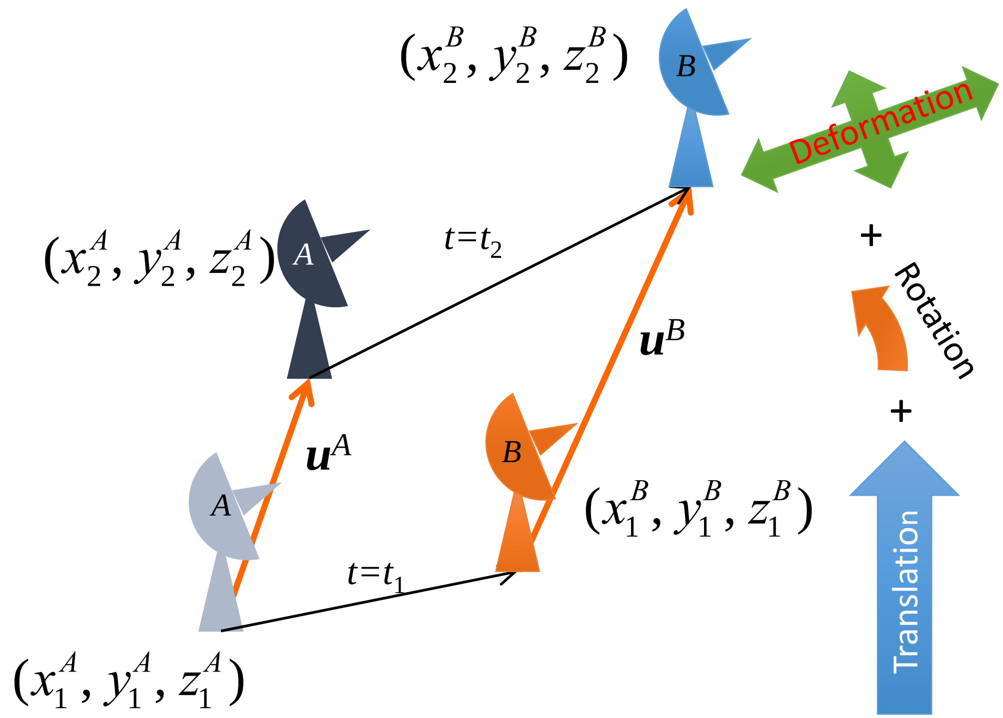
$$u = (x_2 - x_1, y_2 - y_1, z_2 - z_1)$$
$$\text{o } (u_1, u_2, u_3) = (u, v, w)$$

  - Displacement = Translation + Rotation + Deformation

- Deformation (Strain)

$$e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

Note: Different definition from engineering strain

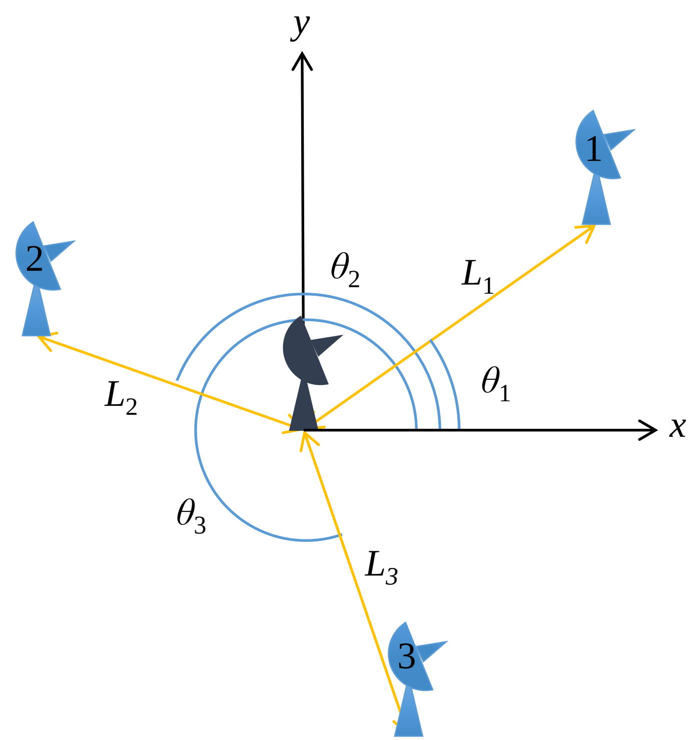


Strain & Principal Strain

- Strain tensor vs change of length

$$\frac{\Delta L_i}{L_i} = \varepsilon_i = \varepsilon_{xx} \cos 2\theta_i + \varepsilon_{yy} \sin 2\theta_i + 2\varepsilon_{xy} \cos\theta_i \sin\theta_i$$

- 3 unknowns
- At least 3 observations are necessary to solve the above equation.
- LSQ are used for more than 4 obs.



Principal Strain

- Principal Strains: Line length change is max/min in the direction of θ, where $\tan 2q = 2e_{xy} / (e_{xx} - e_{yy})$

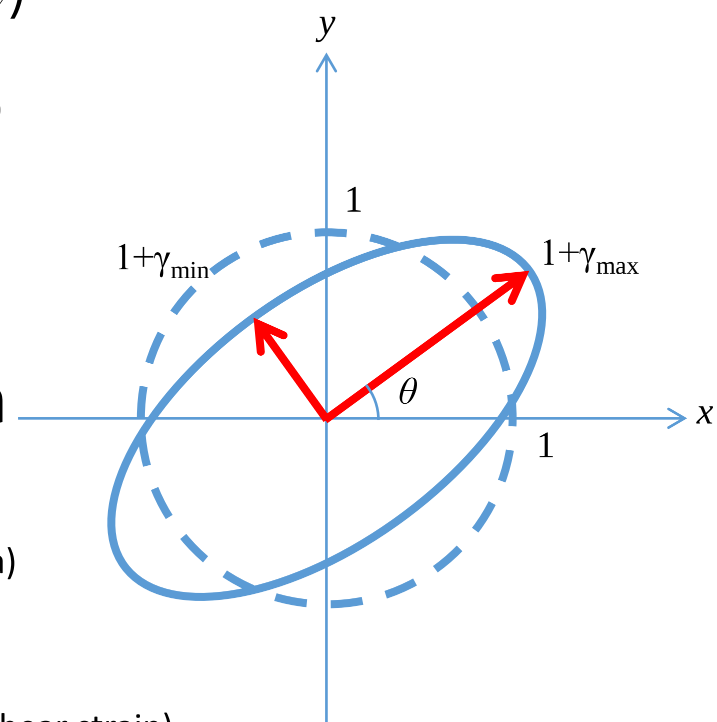
$$g_{max} = \frac{e_{xx} + e_{yy}}{2} + \sqrt{e_{xy}^2 + \frac{(e_{xx} - e_{yy})^2}{4}}$$ (maximum principal strain)

$$g_{min} = \frac{e_{xx} + e_{yy}}{2} - \sqrt{e_{xy}^2 + \frac{(e_{xx} - e_{yy})^2}{4}}$$ (minimum principal strain)

- Dilatation and maximum shear strain

$$Q = e_{xx} + e_{yy} = g_{max} + g_{min}$$ (dilatation = change ratio of area)

$$S = \sqrt{e_{xy}^2 + \frac{(e_{xx} - e_{yy})^2}{4}} = \frac{1}{2}(g_{max} - g_{min})$$ (maximum shear strain)



Strain Ellipse

Shen's (1996) Method

- Computation of displacements and strains at arbitrary points using randomly distributed data
- Relationship between displacement and strain

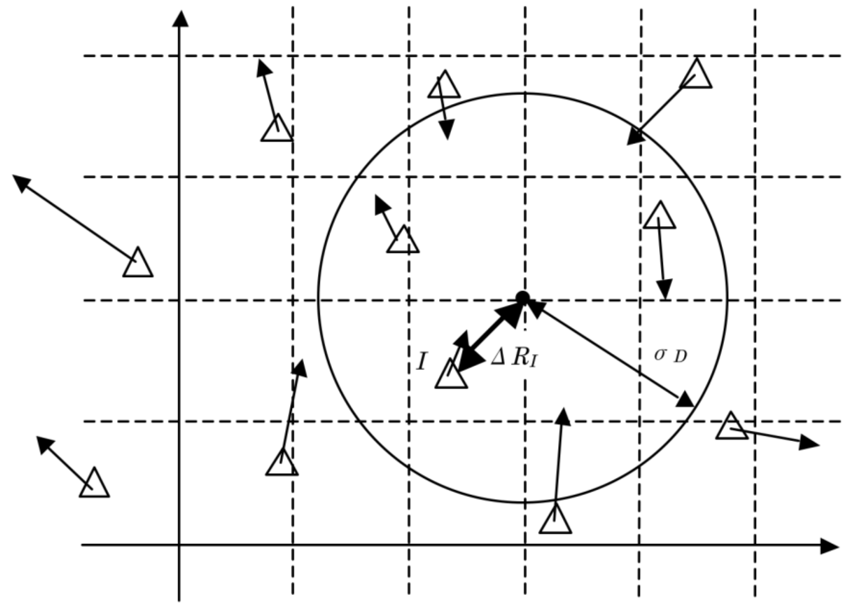
$$u_I = u + e_{xx}\Delta x_I + e_{xy}\Delta y_I + \omega\Delta y_I + \varepsilon_x^I$$

$$v_I = v + e_{xy}\Delta x_I + e_{yy}\Delta y_I - \omega\Delta x_I + \varepsilon_y^I$$

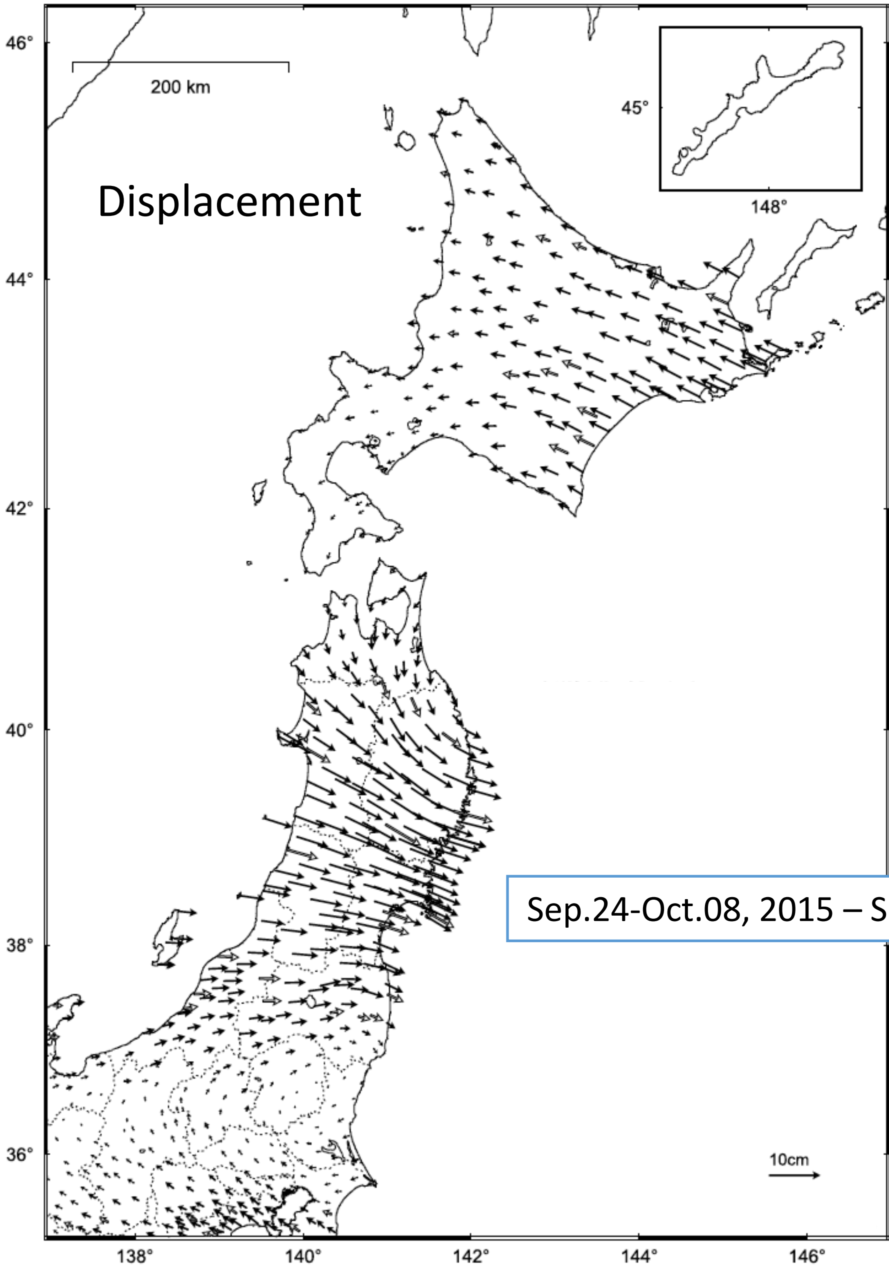
(ω: rotation)

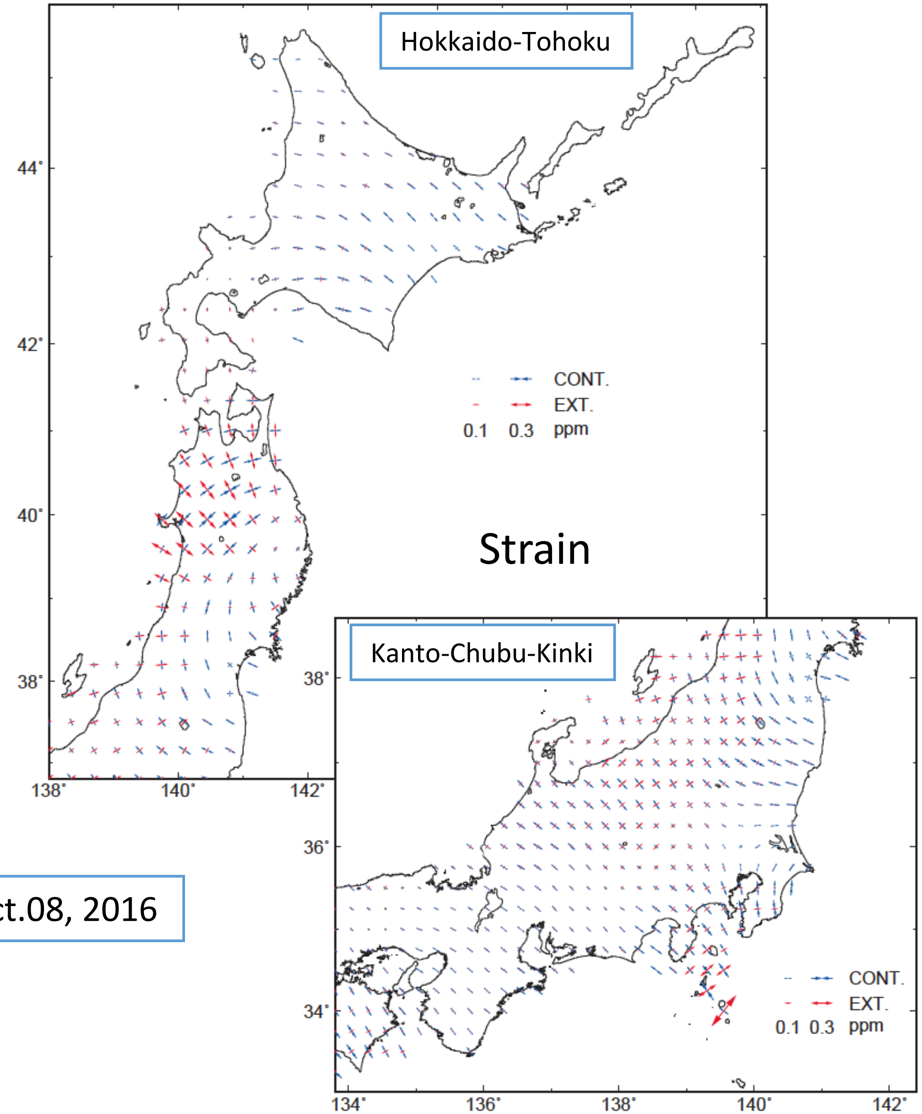
- Weight according to ΔR_I

$$W_{ij} = C_{ij}^{-1} \exp\frac{-\Delta R_I^2}{\sigma_D^2}$$



Observed GPS Velocity vs Principal Strain





Sep.24-Oct.08, 2015 – Sep. 24-Oct.08, 2016

Geographical Information Authority, 2017, Rep. CCEP, 97, 13-23.

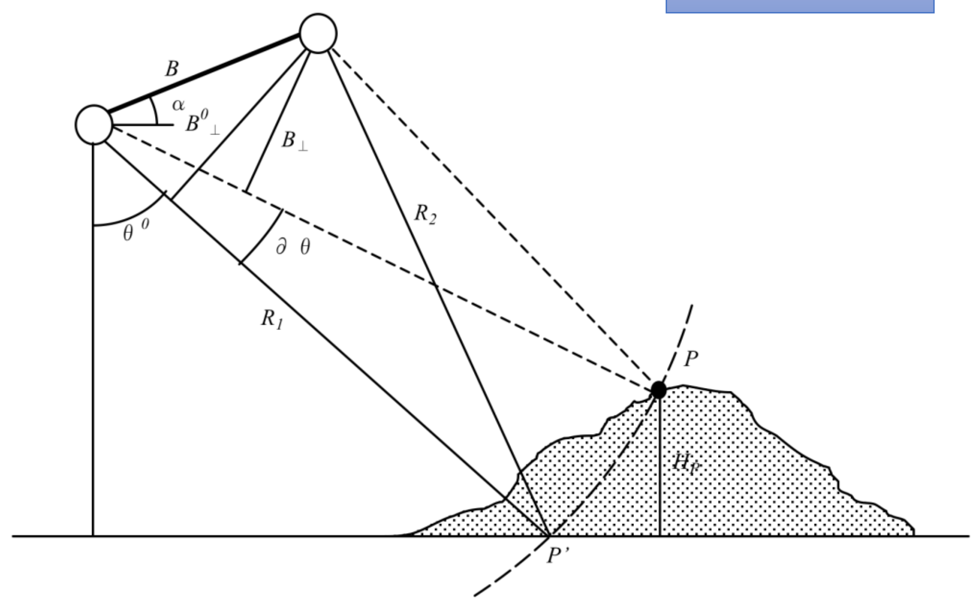
Geometry of SAR Interferometry

- SAR measures distance using phase of microwave.
- Phase difference between two observations contains
  - Difference in orbits
  - Deformation
  - Effect of topography

orbits　　deformation

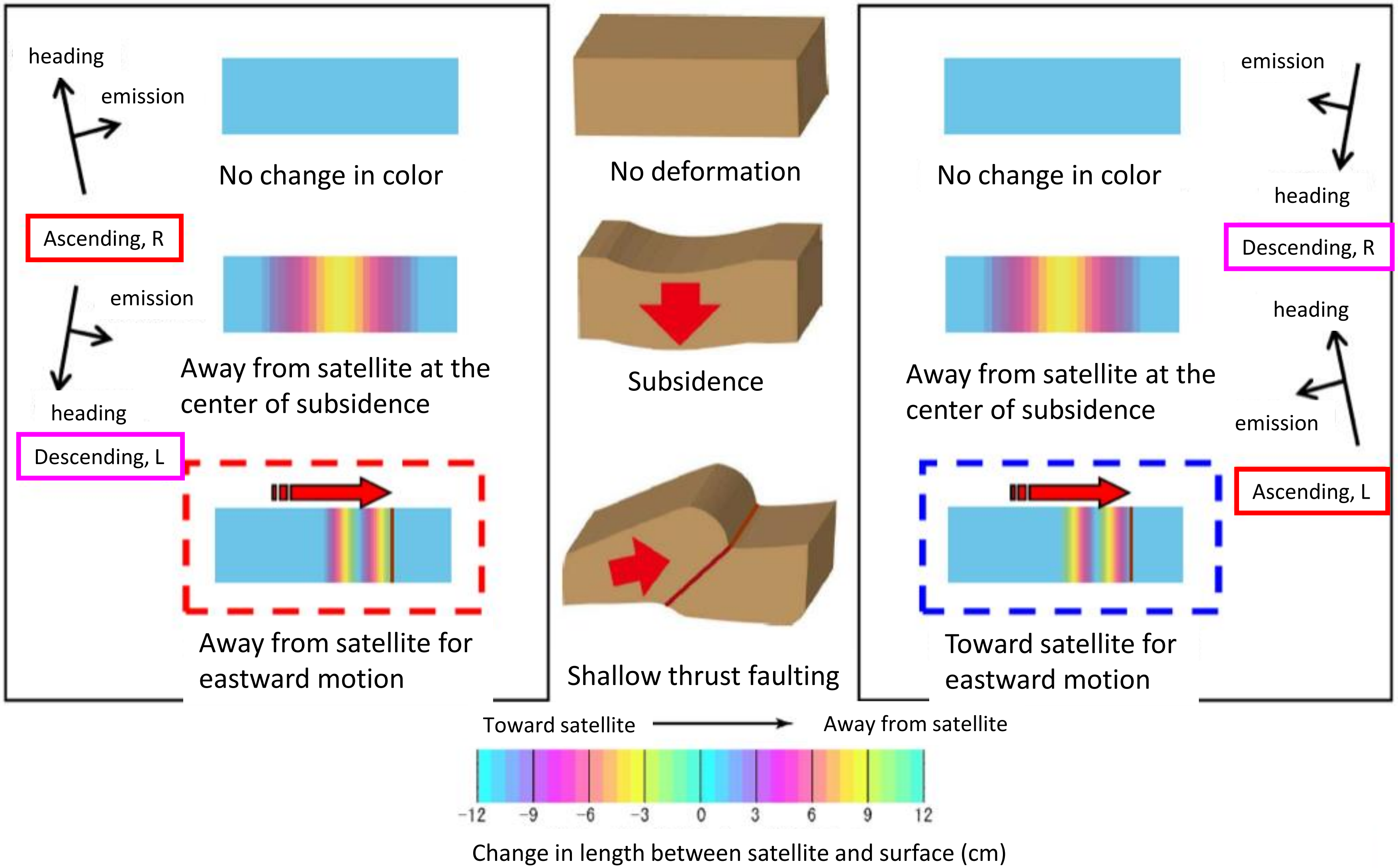
$$\varphi_p = \frac{4\pi}{\lambda}\left(B\sin\left(\theta_p^0 - \alpha\right) - D_p + \frac{B_{\perp,p}^0}{R_{1p}\sin\theta_p^0} H_p \right)$$

topography



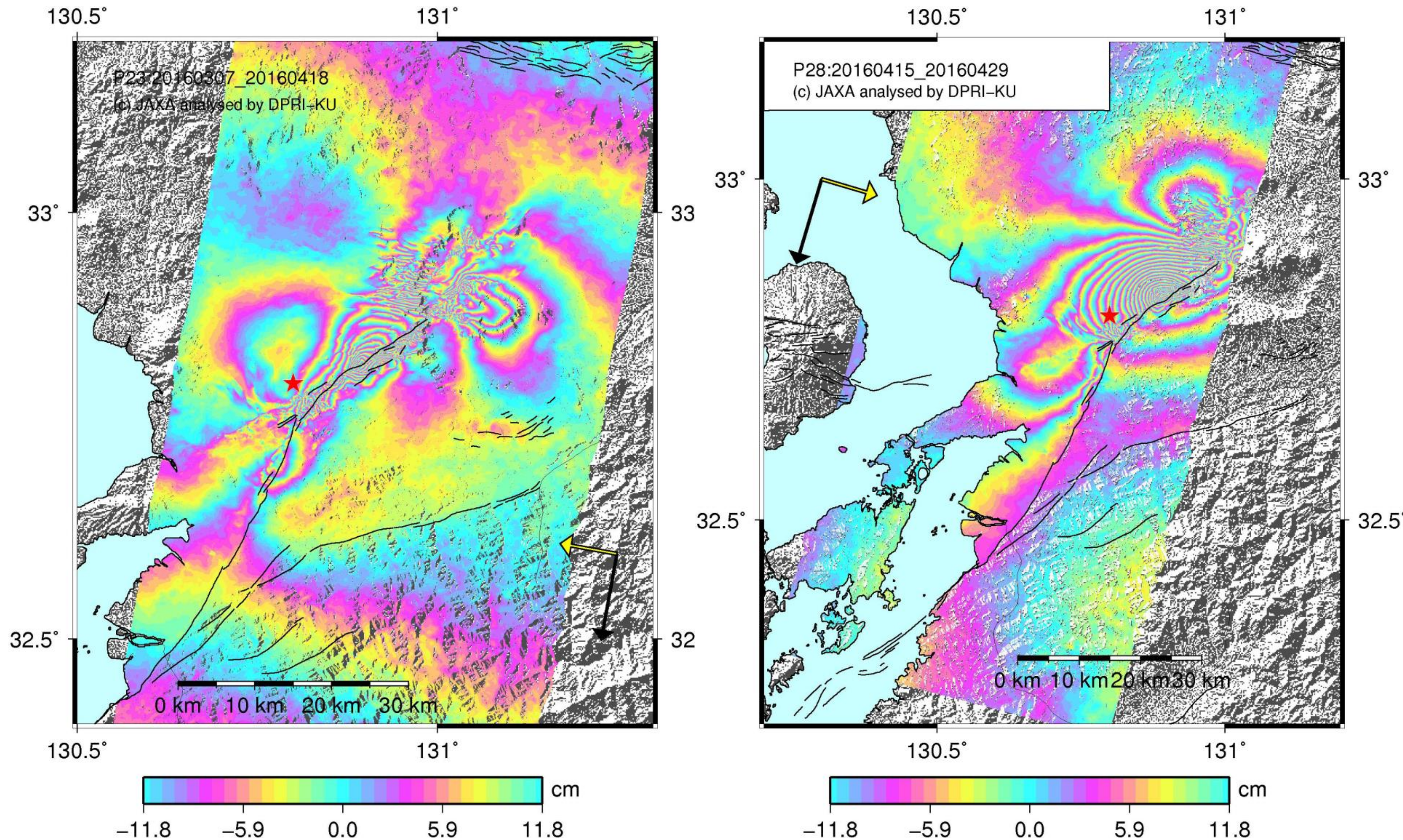
After Hanssen (2001): "Radar Interferometry Data Interpretation and Error Analysis"

Interpretation of Interferogram



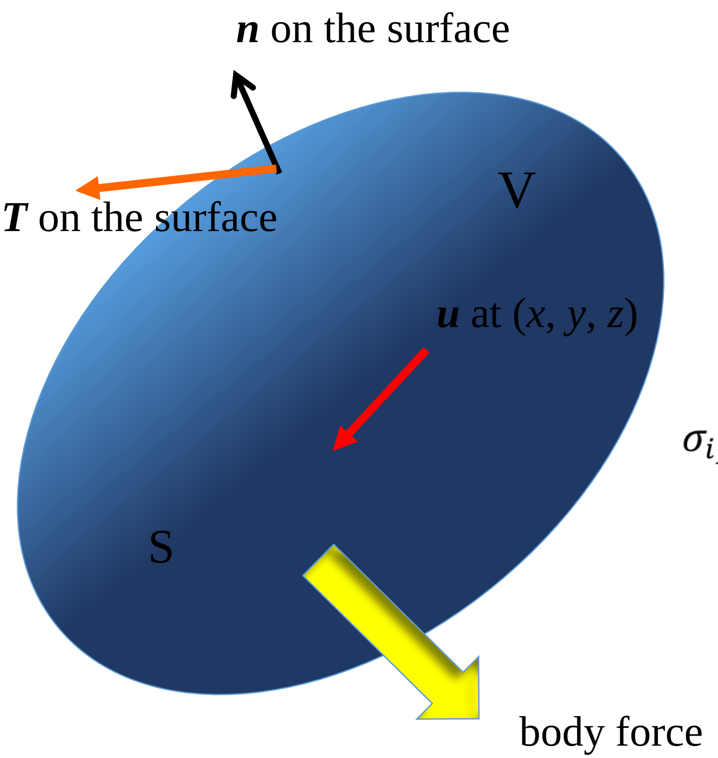
From the website of InSAR, Geospatial Information Authority (as of August, 2015)

Coseismic Interferograms



Dislocation Theory

Basic Equations in Mechanics of Elasticity



$$\rho \ddot{u}_i = f_i + \frac{\partial \sigma_{ij}}{\partial x_j}$$ Equation of Motion

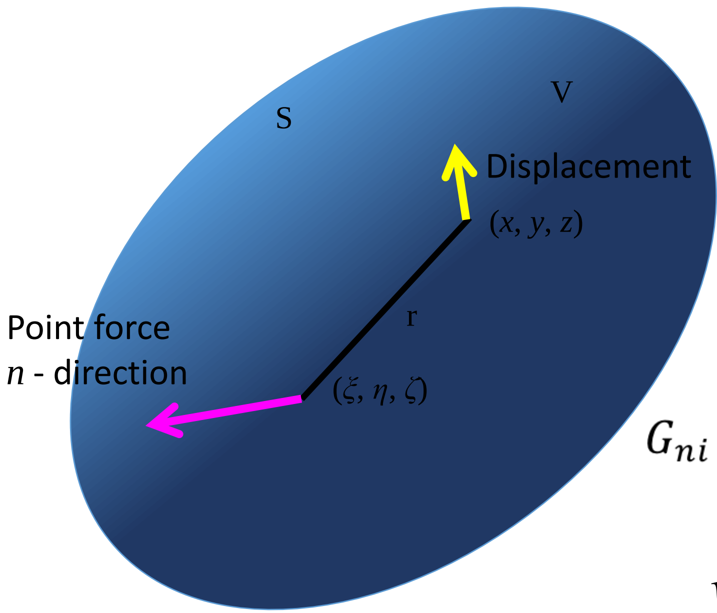
$$T_i = \sigma_{ij} n_j$$ Traction

$$\sigma_{ij} = c_{ijkl} e_{kl} = c_{ijkl} \frac{\partial u_k}{\partial x_l}$$ Hooke's Law

$$e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$ Strain vs Displacement

ρ:density, u_i: *i*-th component (i=1,2,3) of displacement, e_{ij}: *i-j* component of strain
σ_{ij}: *i-j* component of stress, T_i: *i*-th component of traction, f_i: *i*-th component of force
c_{ijkl}: stiffness tensor (elastic constants)

Green's Function for Infinite Homogeneous Isotropic Elastic Body



- Point force in the n – direction (n=1,2,3) at (ξ, η, ζ)
- i – component (i=1,2,3) of displacement at (x, y, z)

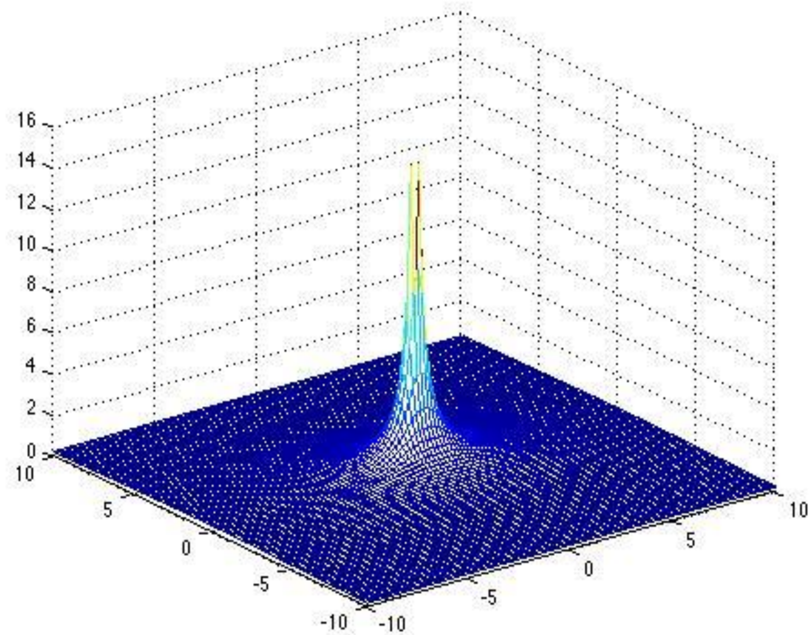
$$G_{ni} = \frac{(\lambda+\mu)}{8\pi\mu(\lambda+2\mu)}\left\{\frac{r_i r_n}{r^3} + \frac{\lambda+3\mu}{\lambda+\mu}\frac{\delta_{ni}}{r}\right\} = \frac{1}{8\pi\mu}\left\{(2-\alpha)\frac{\delta_{ni}}{r} + \alpha\frac{r_n r_i}{r^3}\right\}$$

$$where \quad r_i = x_i - X_i, \quad r = \sqrt{(x-x)^2 + (y-h)^2 + (z-z)^2} \quad a = \frac{1}{2(1-n)}$$

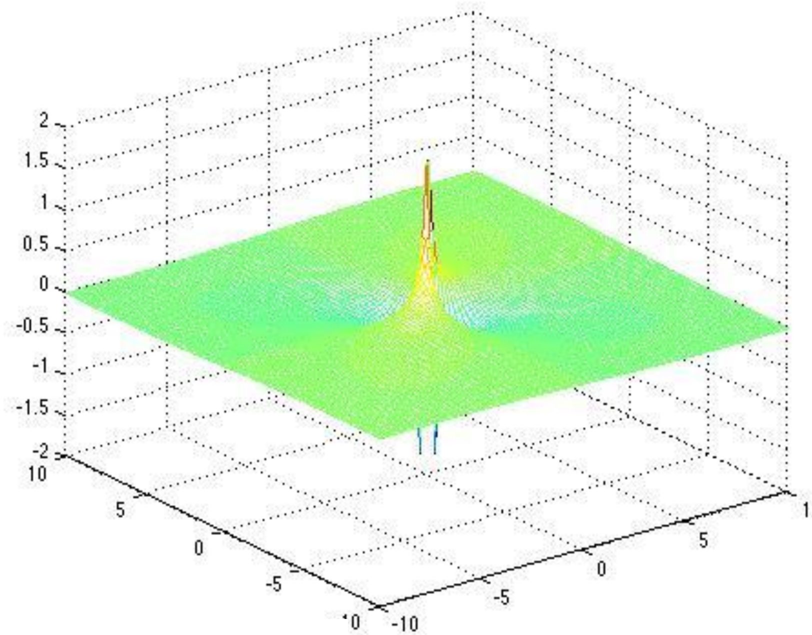
λ, μ: Lame's constants, v: Poisson's ratio, δ_{ni}: *Kronecker Delta* (n,i=1,2,3)

- Displacement decays with $1/r$

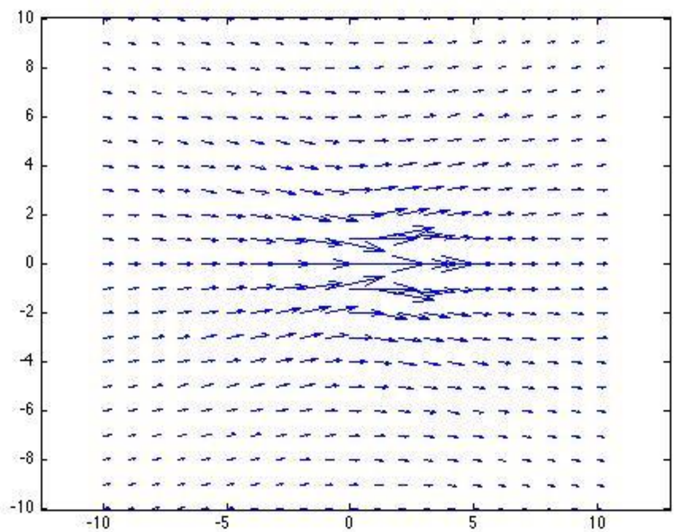
Green's Function: *G*11 and *G*12



*G*11 at z=0

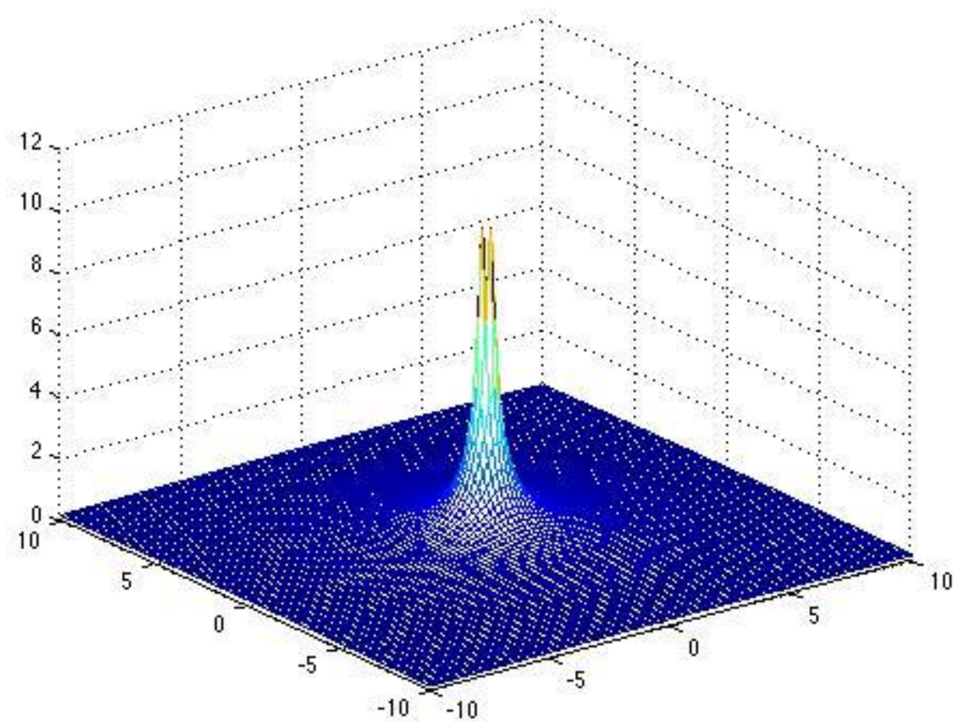


*G*12 at z=0

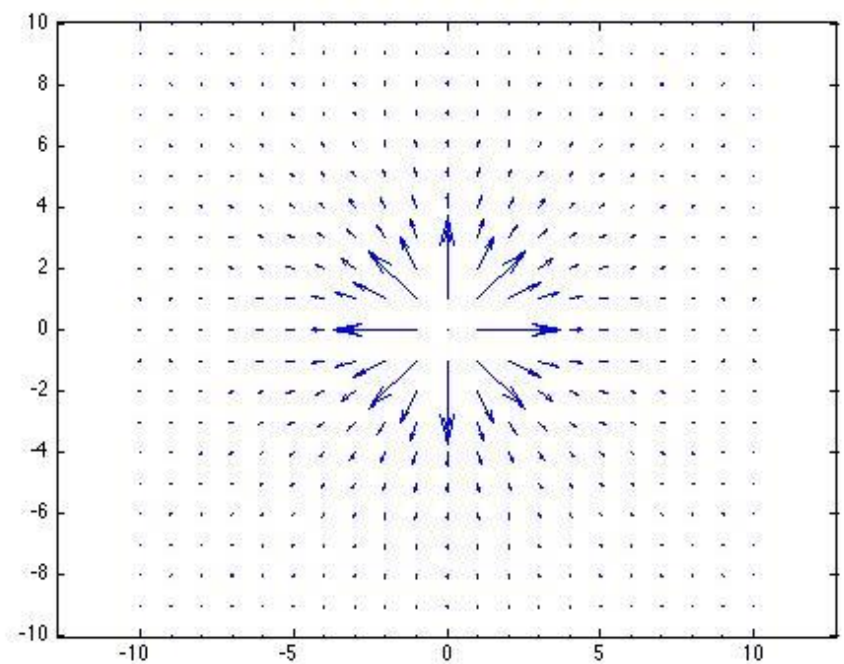


Horizontal vector for *G*11 and G12 at z=0

Green's Function: *G3i*



*G*33 at z=0



Horizontal vector for *G*31 and G32 at z=1

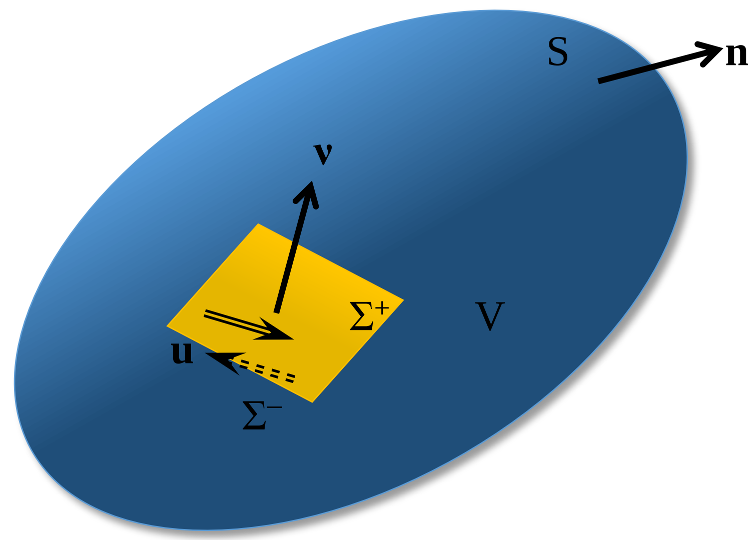
Dislocation Theory: Volterra's Formula

silp

Green's function

$$u_m(x) = \iint_{\Sigma} [u_i(\xi)]\, c_{ijkl} \frac{\partial G_{mk}(x;\xi)}{\partial \xi_l} n_j \, d\Sigma(\xi)$$

Fault surface

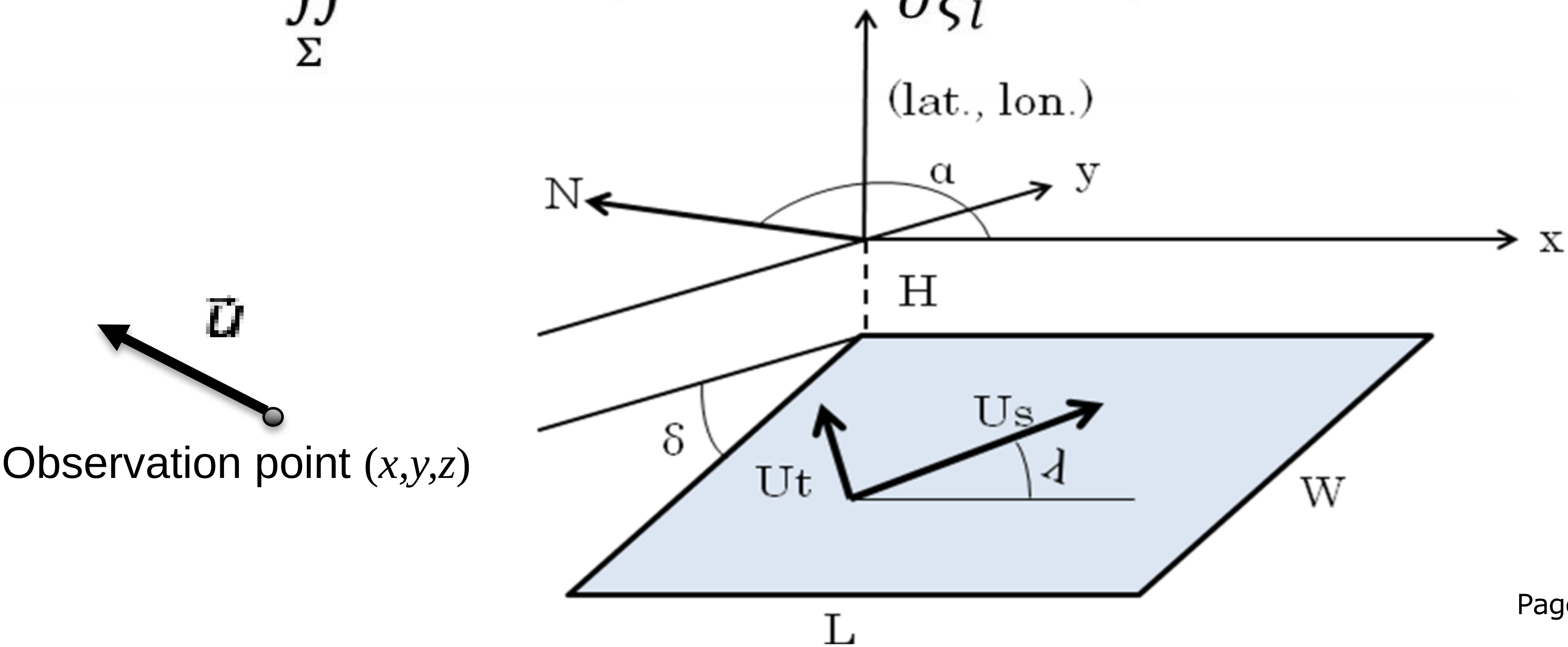


- Displacements are uniquely determined with displacement discontinuity (= slip) on a given internal surface in an elastic body.
- Displacement decays with $1/r^2$

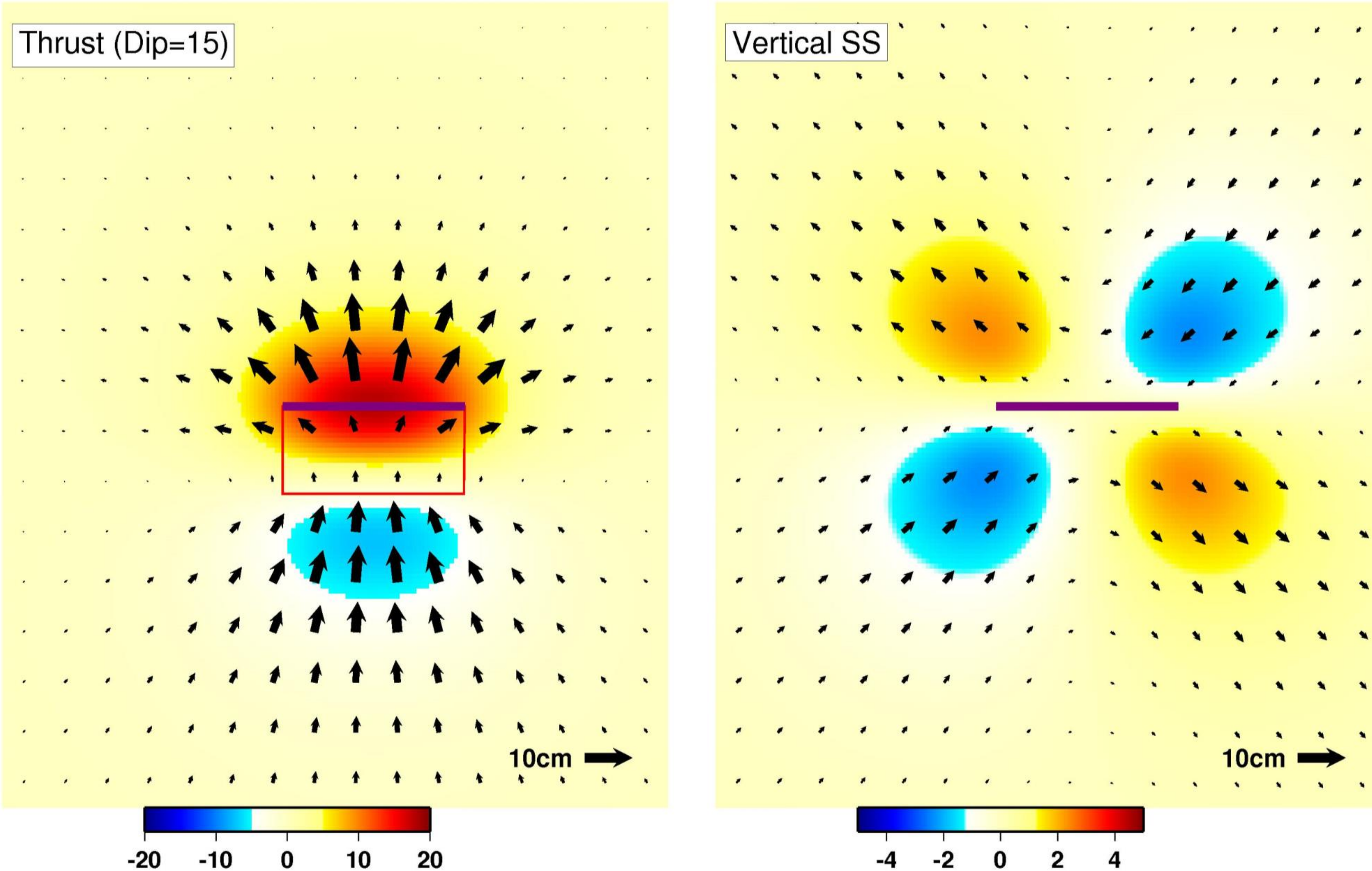
Geometry of Fault and Observation Point

- Fault model: We can compute deformation and shaking by knowing the geometry, location and slip of fault in the earth.

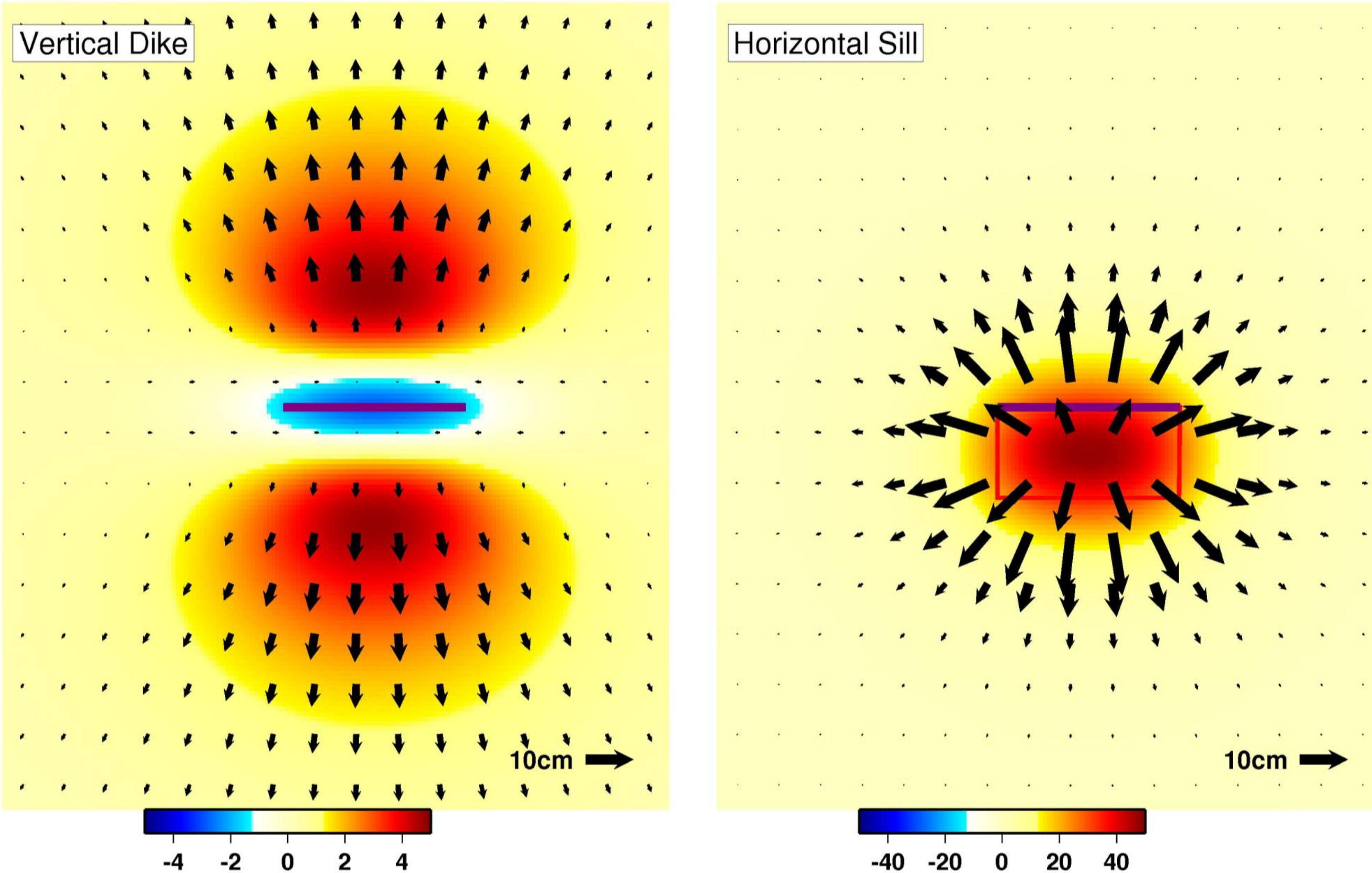
$$u_n(\mathbf{x}) = \iint_{\Sigma} [u_i(\boldsymbol{\xi})] c_{ijkl}(\boldsymbol{\xi}) \frac{\partial G_{nk}(\mathbf{x};\boldsymbol{\xi})}{\partial \xi_l} \nu_j \, d\Sigma(\boldsymbol{\xi})$$



Basic Pattern of Deformation: Thrust and Strike-Slip



Basic Pattern of Deformation: Dike and Sill



Inversion of Slip

- Volterra's formula gives relationship between displacements and faulting (slip, geometry).

$$u = f(x, y, z; Lon, Lat, \varphi, H, L, W, \delta, \lambda, slip) + \epsilon$$

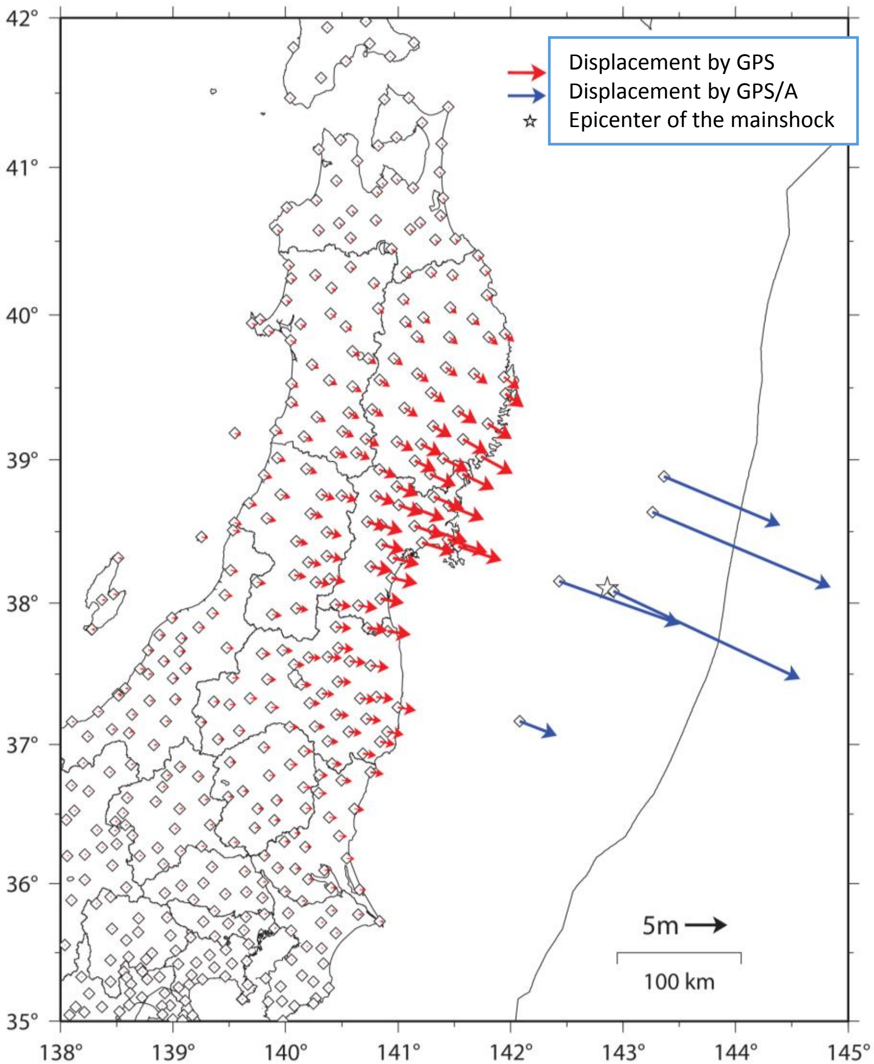
- If we know/assume geometry of fault(s), displacement is linearly dependent on slip (s).

$$u = G(x; X)s + \epsilon$$

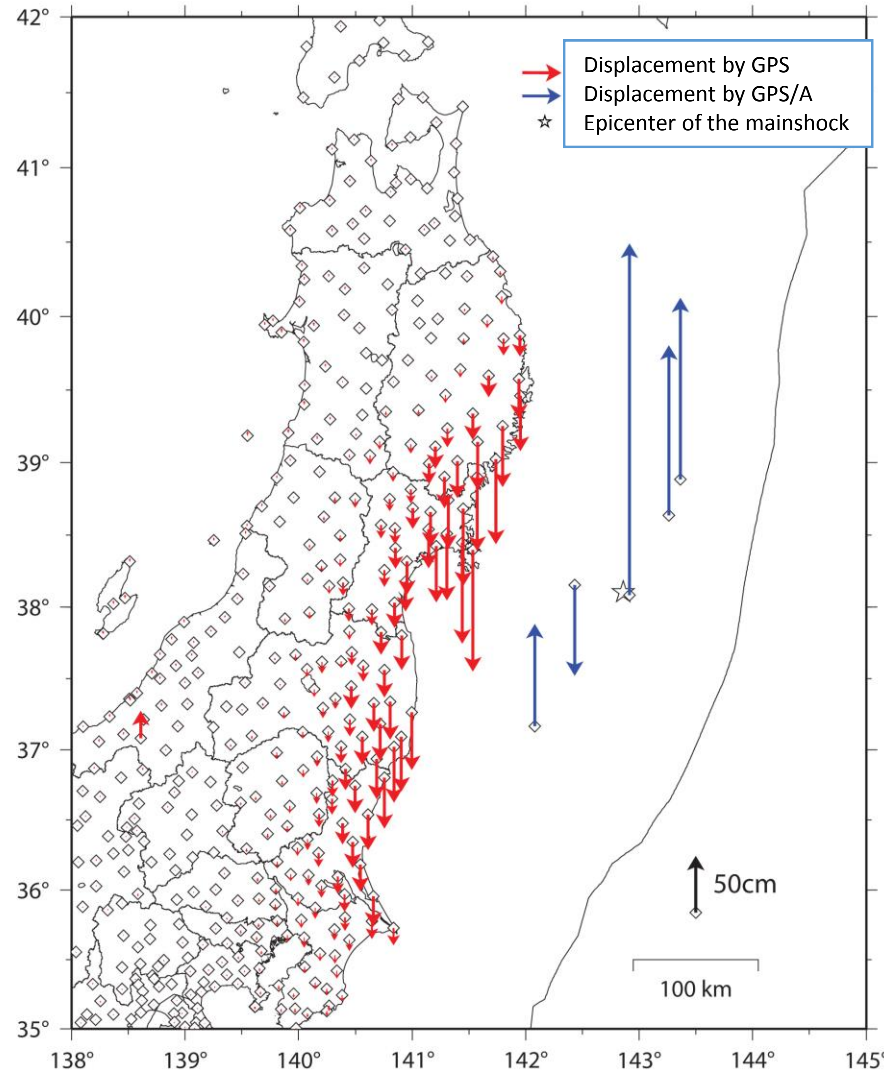
Observation error

- We can estimate slip by solving this equation with LSQ etc.
- *a priori* constraints (smooth distribution, fixed direction of slip etc.) are usually applied.

2011 Tohoku Earthquake



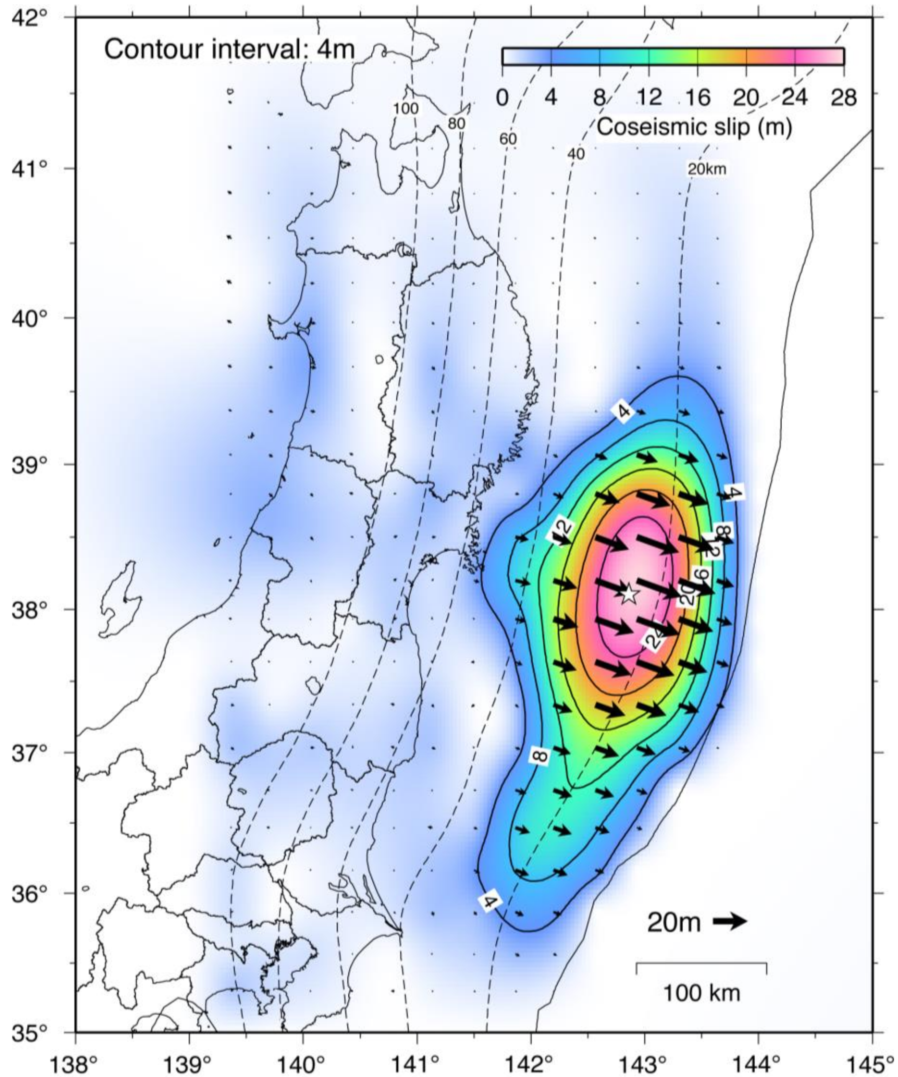
Horizontal Displacement



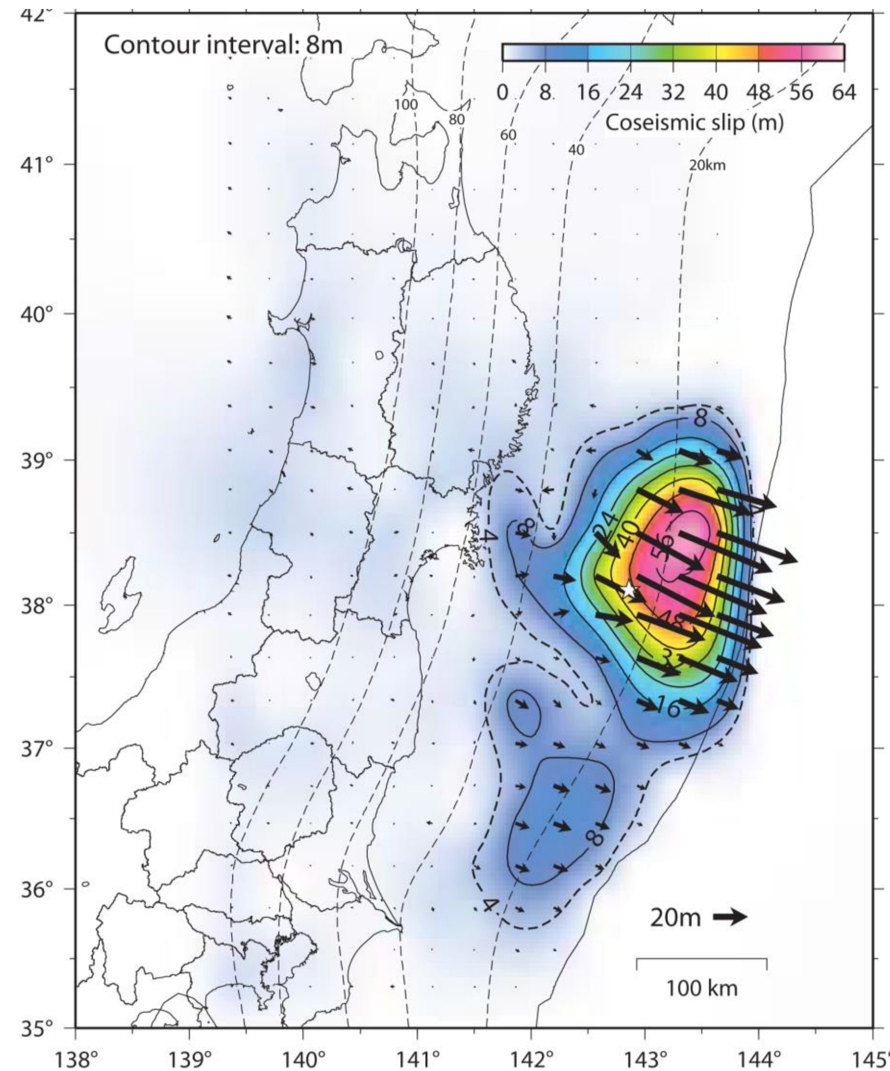
Vertical Displacement

Geographical Information Authority, 2011, Rep. CCEP, 86, 184-272.

Estimate Slip by GSI (2011)



Without displacements by GPS/A



With displacements by GPS/A

Geographical Information Authority, 2011, Rep. CCEP, 86, 184-272.

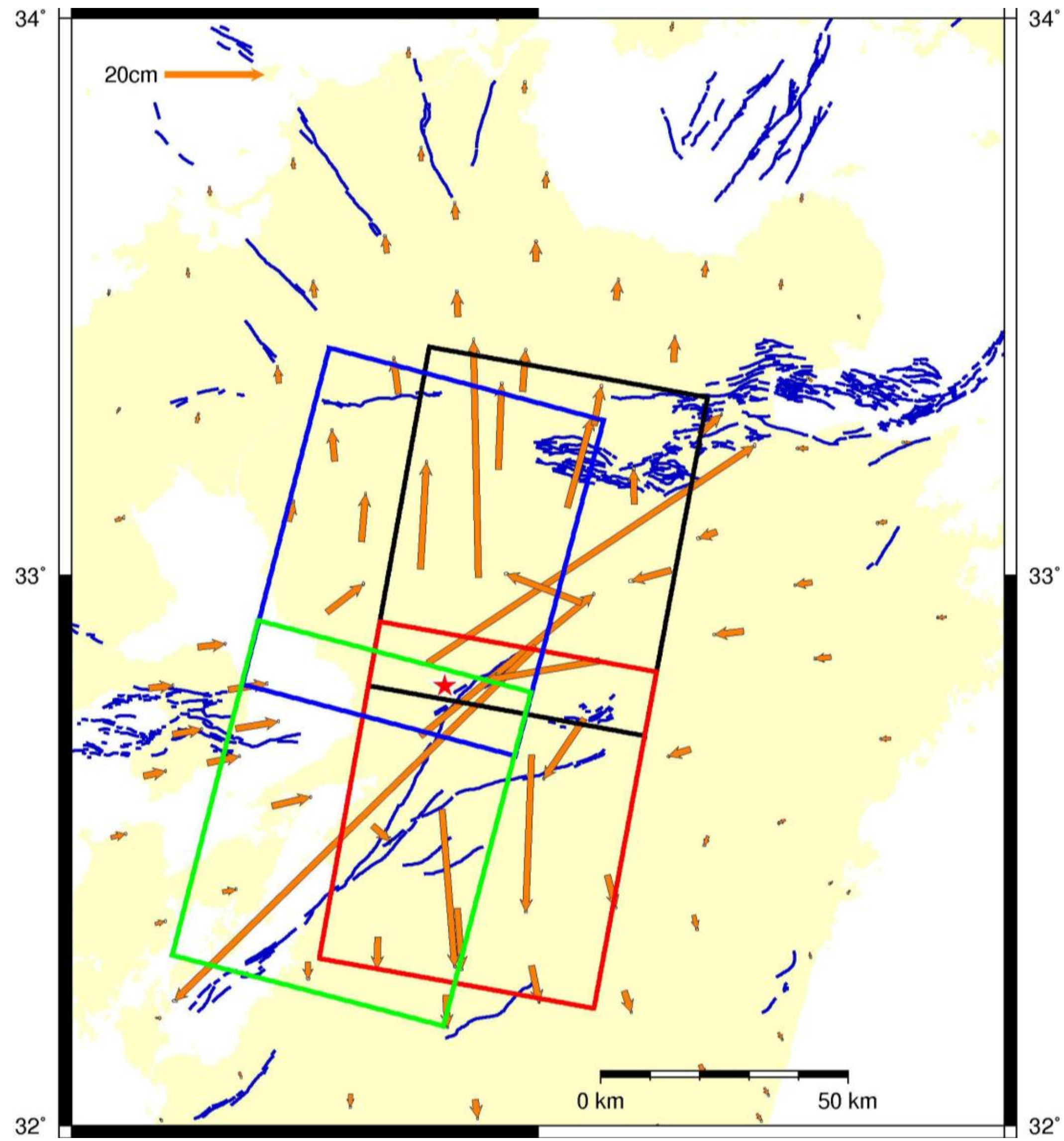
Recent Typical Examples

2016 Kumamoto earthquake

2015 Sakurajima crisis

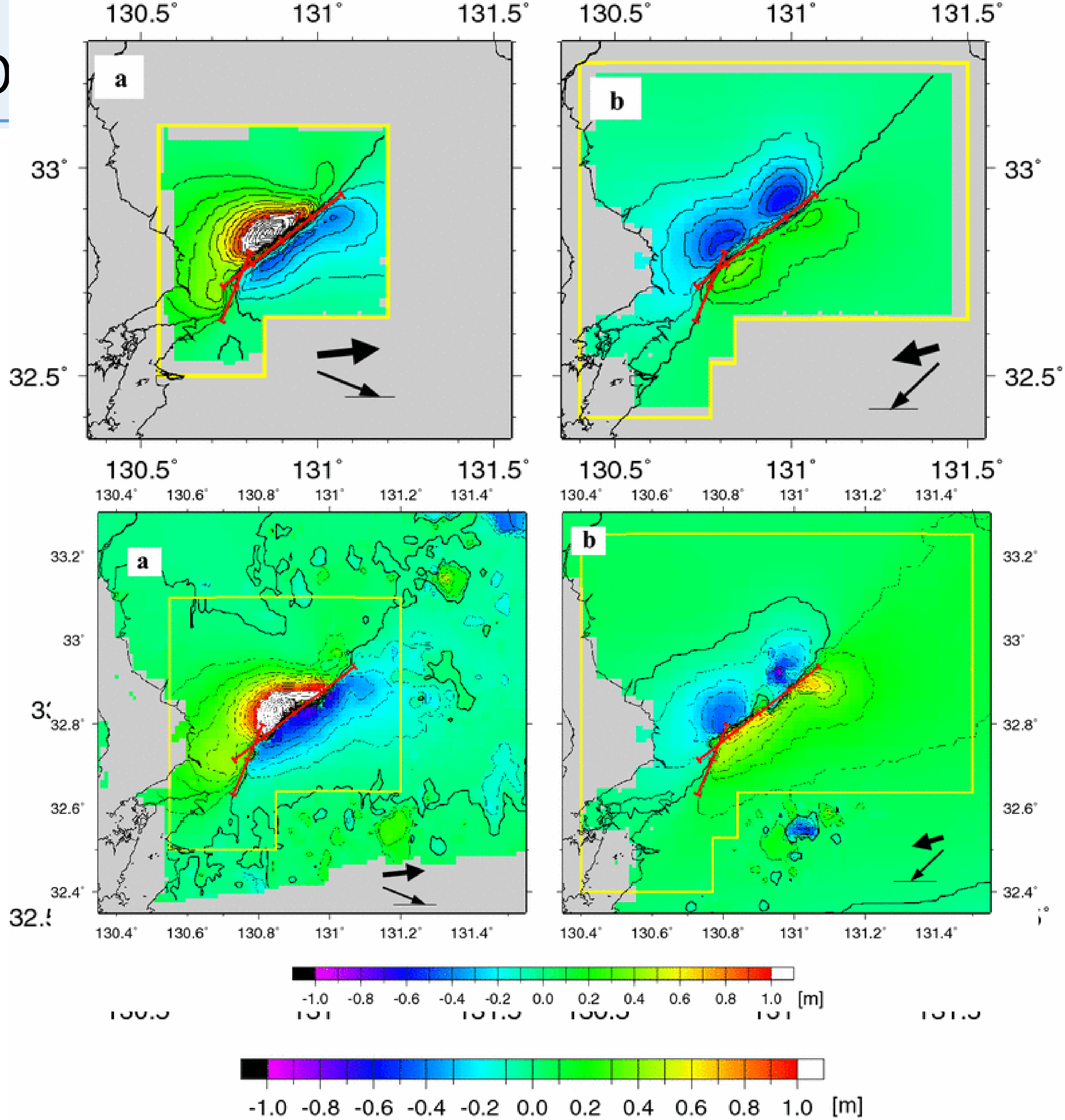
2016 Kumamoto Earthquake

- Displacement vectors based on F3 solution of GEONET (GSI, 2016)
- Max 1 m displacements along the Futagawa Fault
  - North side to the east-north, while south side to the west-south.
  - Right-lateral strike slip along NE-SW trending faults



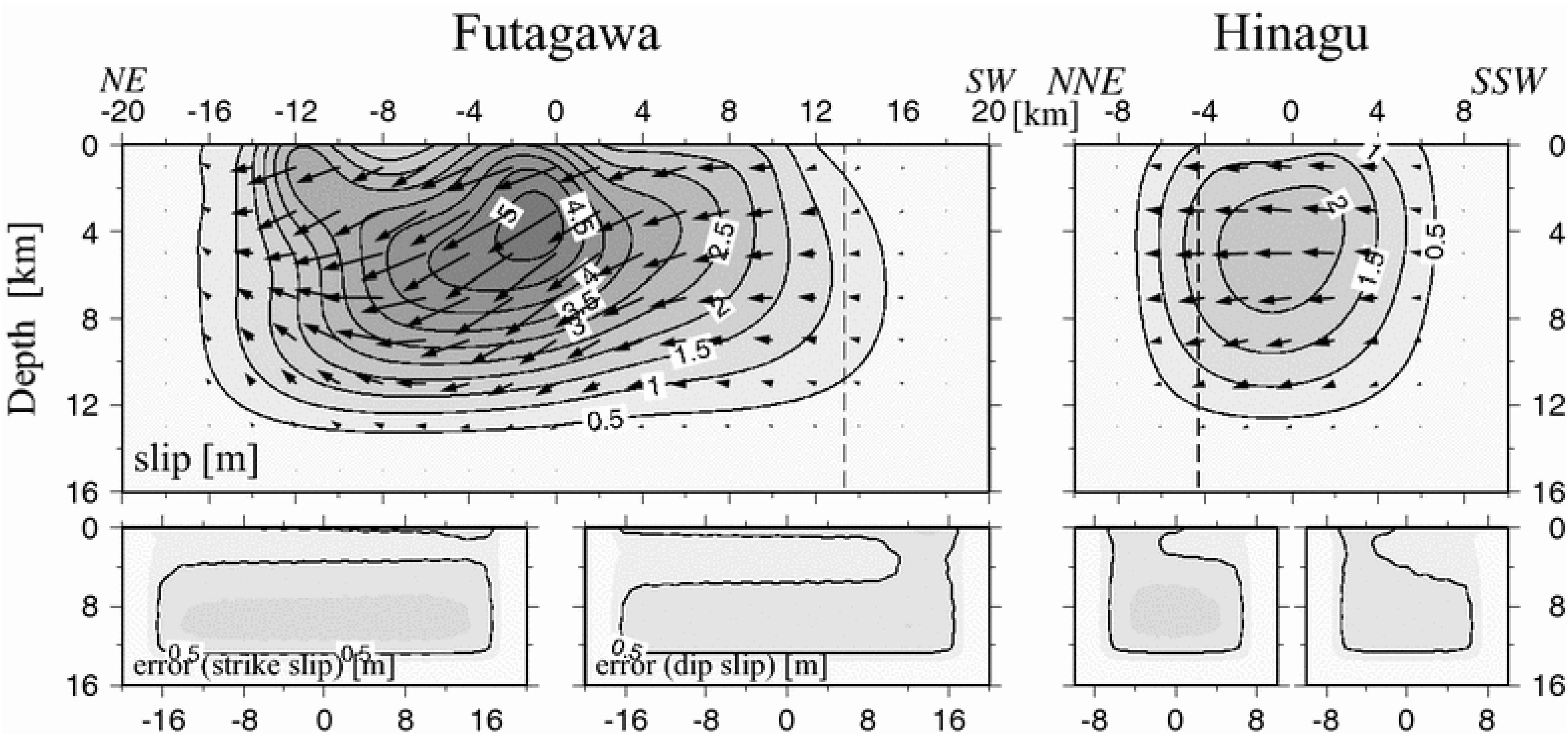
Inversion of Slip

- Estimate of slip and dip angle from observed interferograms
- 2 fault planes
- Distributed slip



Fukahata and Hashimoto, 2016, EPS, 68:204, DOI:10.1186/s40623-016-0580-4

Estimated Slip Distribution



Fukahata and Hashimoto, 2016, EPS, 68:204, DOI:10.1186/s40623-016-0580-4

2015 Crisis of Sakurajima Volcano

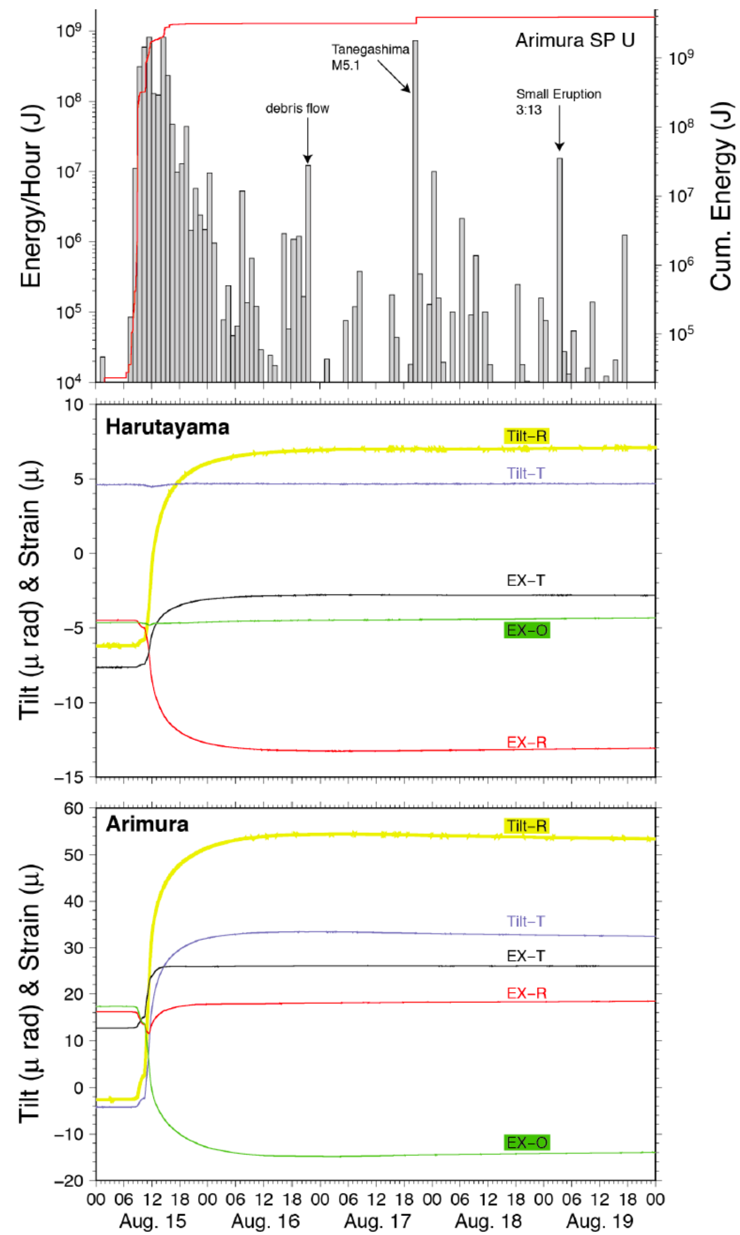
Photo from the website of Sakurajima Volcano Observatory, DPRI, Kyoto Univ.,
http://www.svo.dpri.kyoto-u.ac.jp/svo/?page_id=69

Seismicity and Deformation during August, 2015, Crisis

- On Aug. 15, sudden increase of seismicity
- Rapid change in strain and tilt up to 6 x 10^{-5} during < 4 hours
- JMA issued level 4 alert

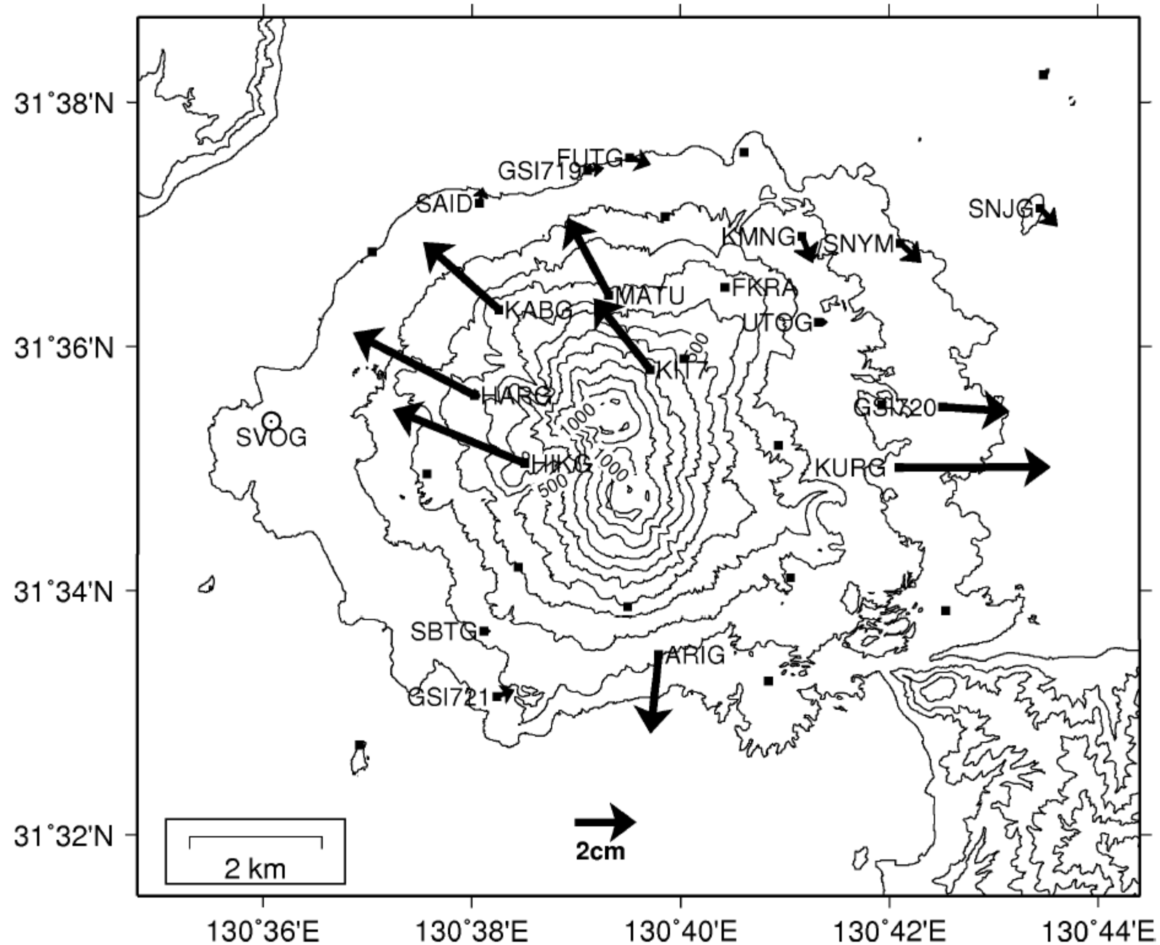
SVO, Kyoto University (2015), Presented in CCPVE on Aug. 21, 2015

http://www.data.jma.go.jp/svd/vois/data/tokyo/STOCK/kaisetsu/CCPVE/shiryo/kakudai150821/6_kyodai_sakurajima.pdf



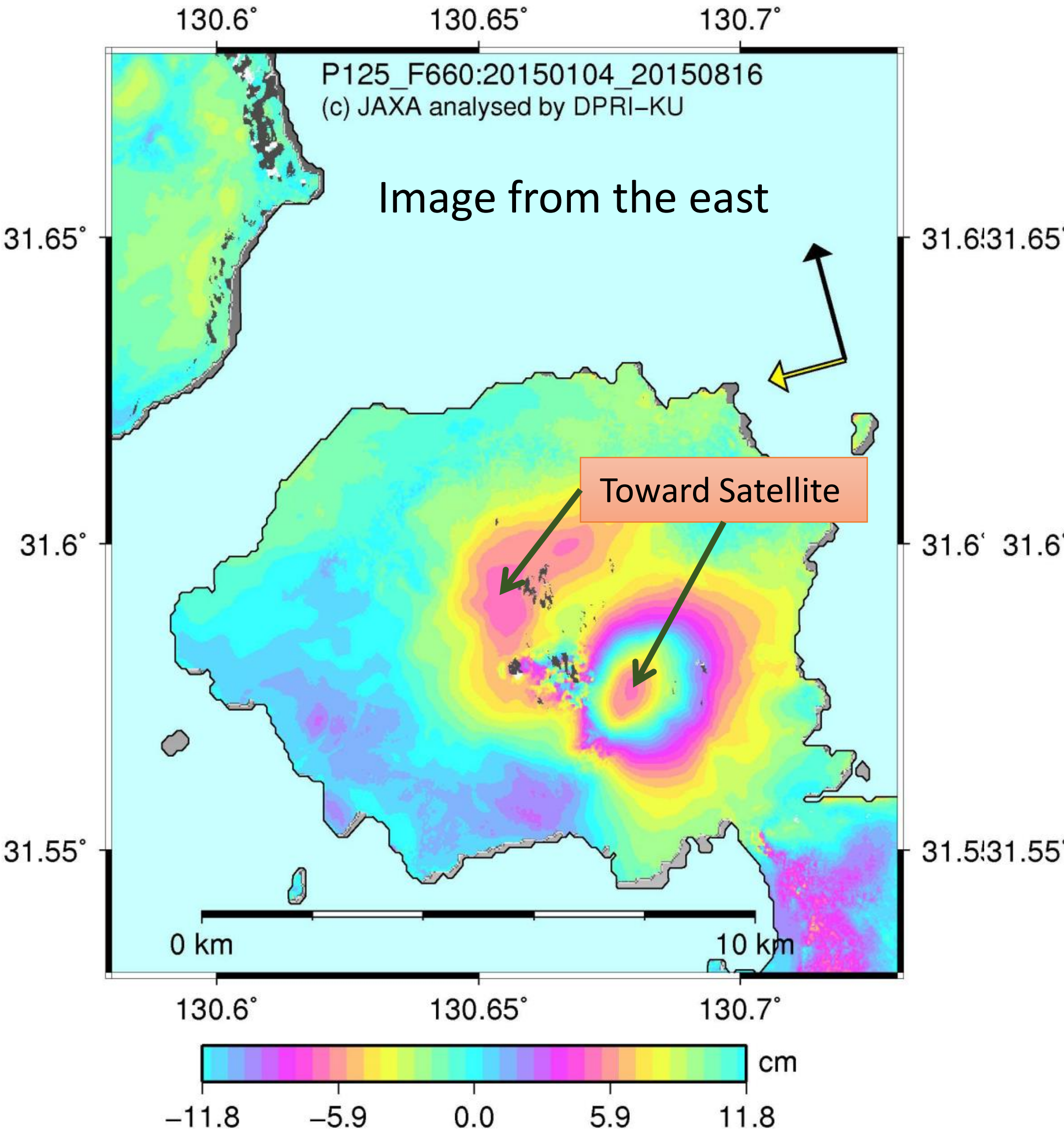
Horizontal Displacement Observed by GPS

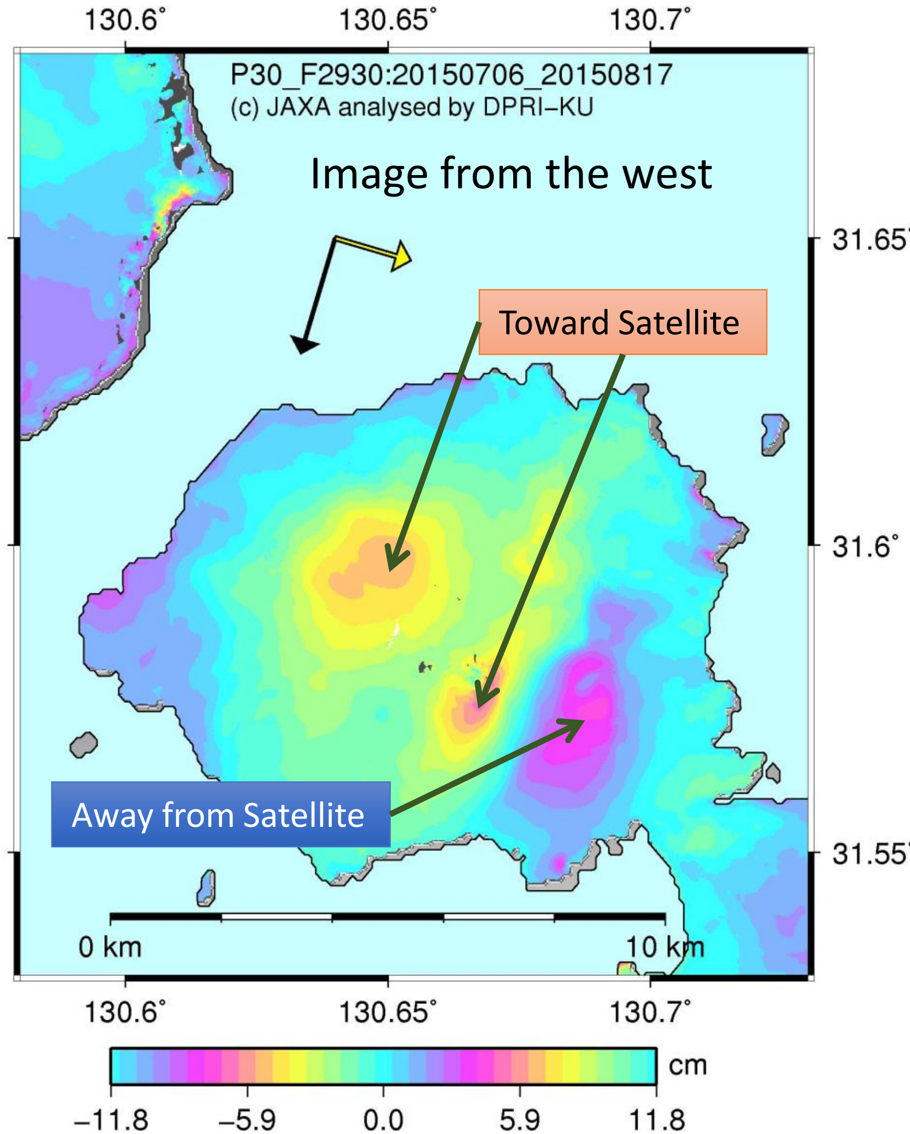
- Western side of volcano moved to the west.
- Eastern side moved to the east.



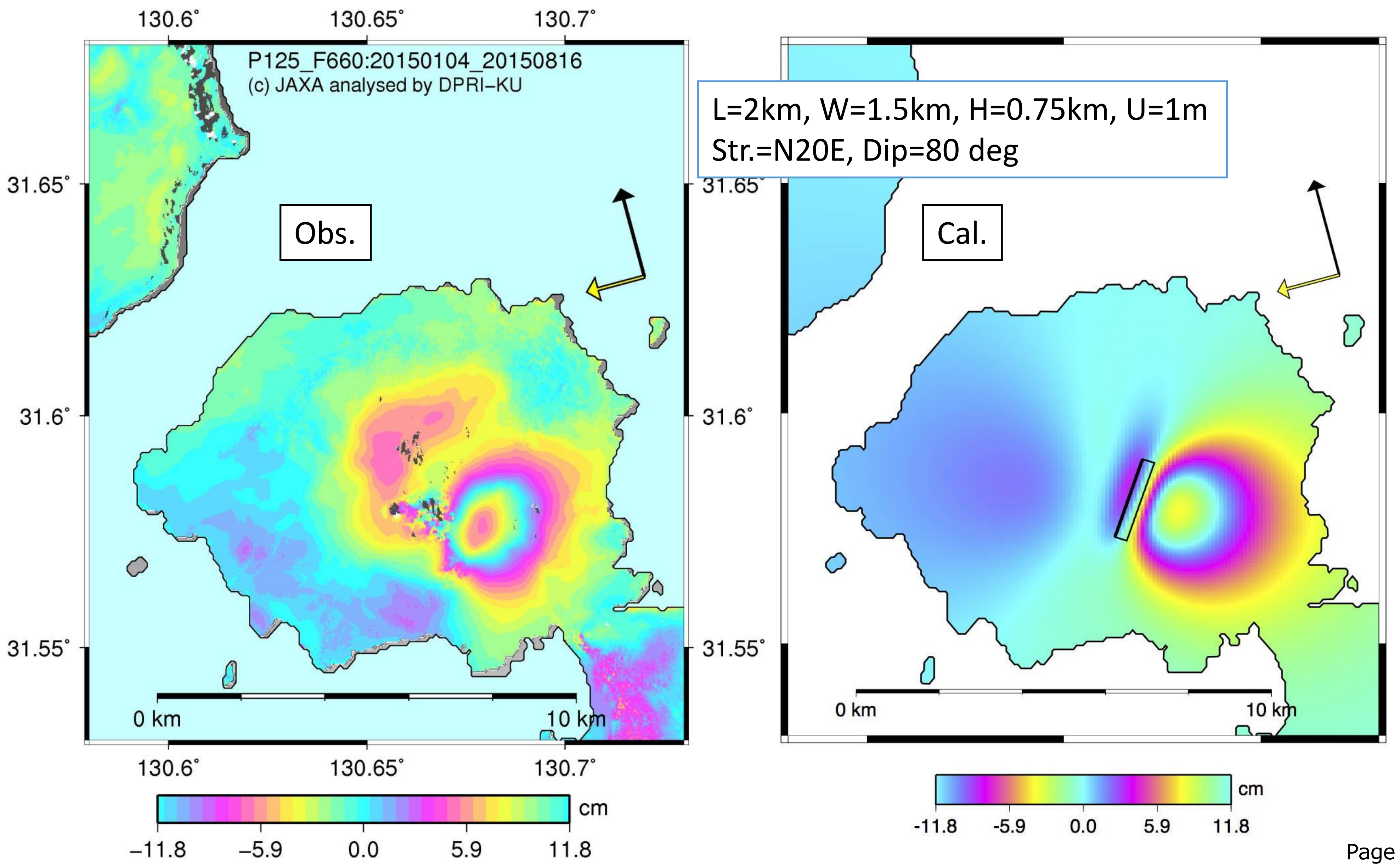
SVO, Kyoto University (2015), Presented in CCPVE on Aug. 21, 2015
http://www.data.jma.go.jp/svd/vois/data/tokyo/STOCK/kaisetsu/CCPVE/shiryo/kakudai150821/6_kyodai_sakurajima.pdf

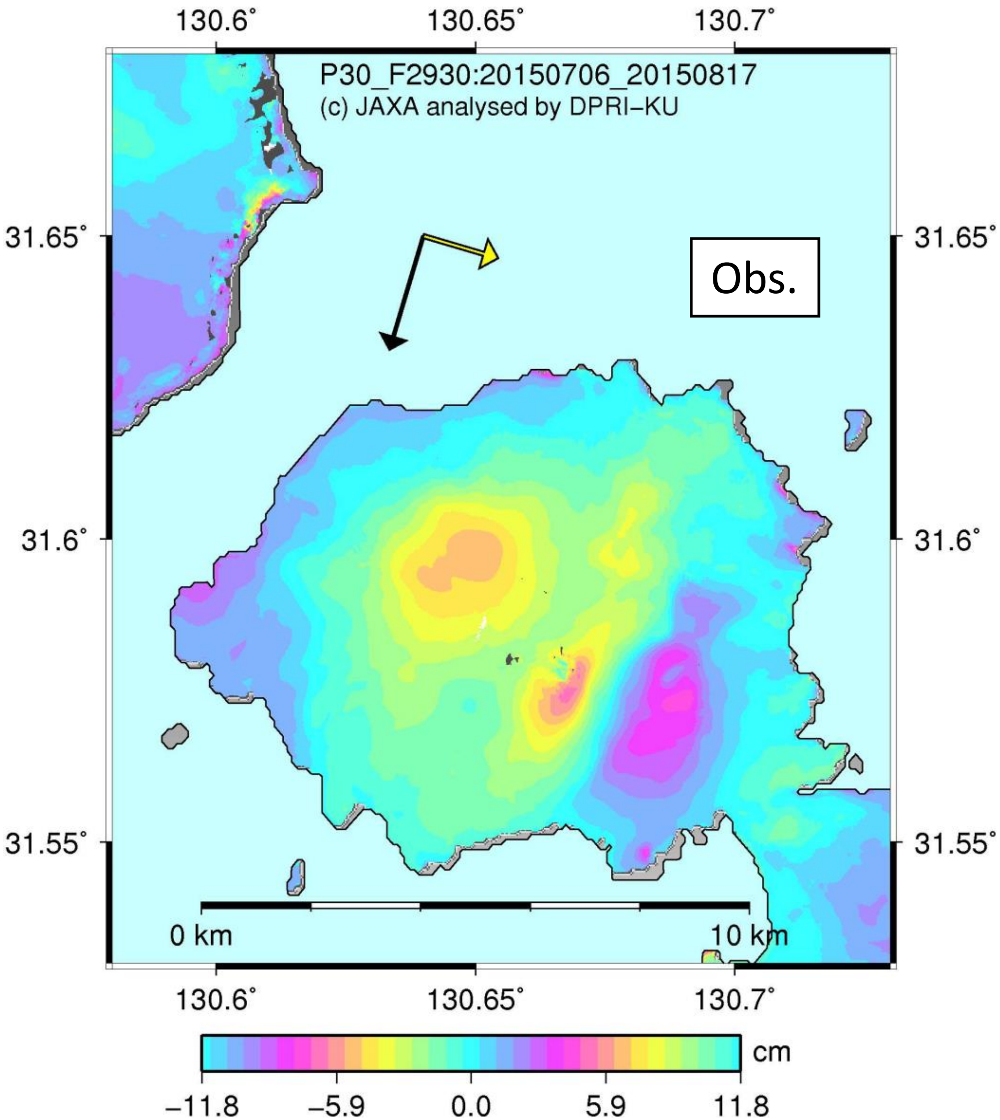
Interferograms

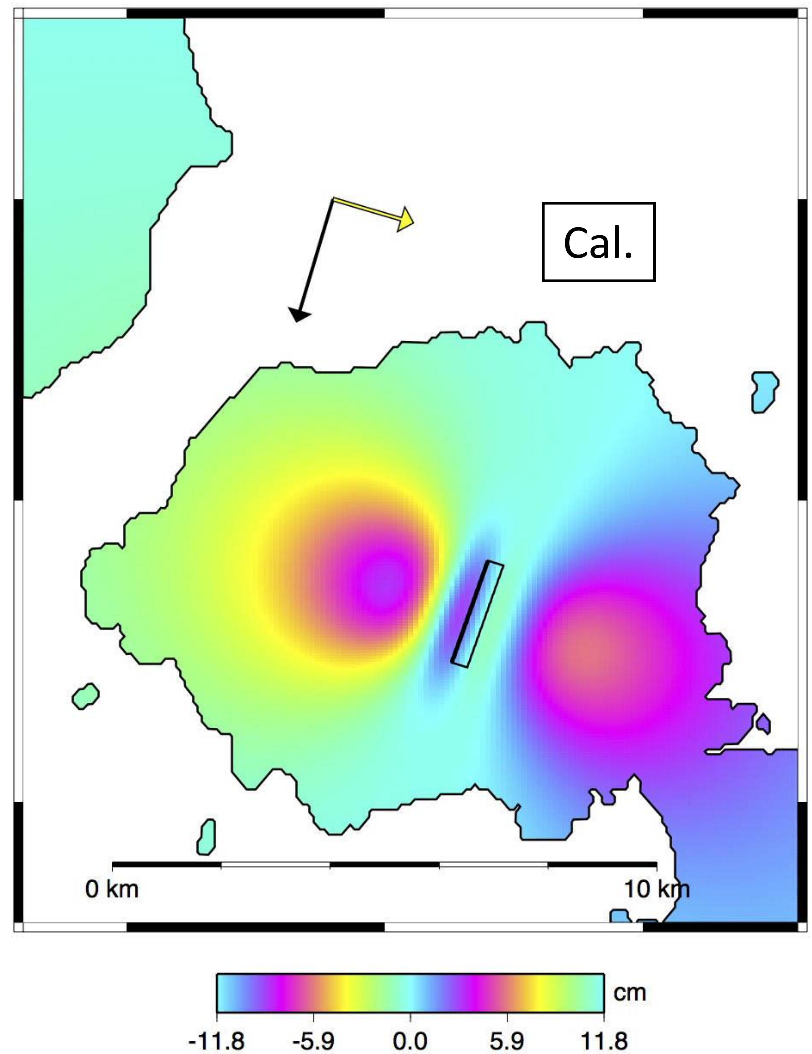




Comparison with Dike Model




130.6˚
130.65˚
130.7˚
P30_F2930:20150706_20150817
(c) JAXA analysed by DPRI−KU
Obs.
31.65˚
31.6˚
31.55˚
0 km
10 km
cm
−11.8
−5.9
0.0
5.9
11.8


Cal.
0 km
10 km
cm
-11.8
-5.9
0.0
5.9
11.8

Summary

- Geodetic surveys provide invaluable data on the deformation of earth's surface.
- We can interpret observed deformation with dislocation model etc. developed from the theory of elasticity.
- *Basic information on the evaluation of activity of earthquakes and volcanoes*
- Both geodesy and theory of elasticity contribute to deepening of understanding of the earth!