Vertical displacements of engineering structures during reconstruction: classification and prediction models

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Abstract

The paper presents the results of research of various classification models for vertical displacements and their further prediction. When performing geodetic monitoring of complex engineering structures under reconstruction, the displacements of the structure is always unequally. In this case, it is impossible to construct a single prediction kinematic model for the whole structure. Therefore, it is necessary to classify the displacements and divide the structure into blocks, within which displacements occur equally. In this paper, two methods for constructing blocks of equal displacements were researched: statistical method and cluster analysis method. Based on the results of geodetic monitoring, it is shown that the statistical method does not allow extracting blocks of equal displacements. Reliable and unambiguous results were obtained using cluster analysis of displacements.
After obtaining blocks within which the process of displacement can be considered uniform, the different prediction kinematic models of displacements by the method of random functions were investigated. A necessary condition for the application of the theory of random functions is the normal distribution of the measured displacements. Verification of displacements by the normal distribution was performed using the skewness and kurtosis criteria. After establishing the normal distribution of displacements, different prediction kinematic models were investigated. It is established that the most adequate prediction kinematic models are third-power polynomial models and piecewise linear models. For error prediction, it is advisable to use polynomial models or logistic functions.

Key words: vertical displacement, block of equal displacements, cluster analysis, random functions, prediction kinematic model, trend approximation, autocorrelation function.

1 DESCRIPTION OF RESEARCH OBJECT

The basis of the presented research were the results of geodetic monitoring of the facade of the historic building in Kiev (see Fig. 1 and Fig. 2). This historical building was built in 1930 in the architectural style of this façade is "socialist realism".

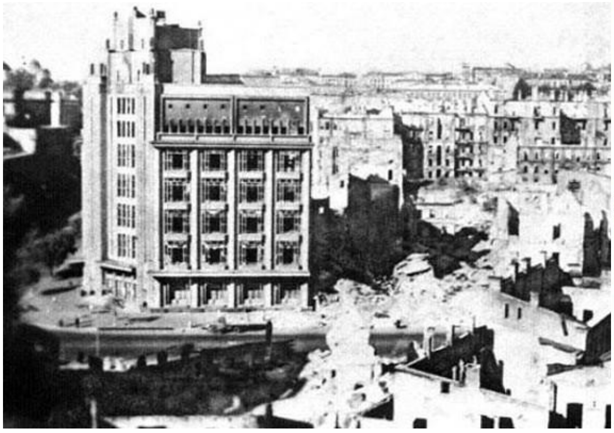
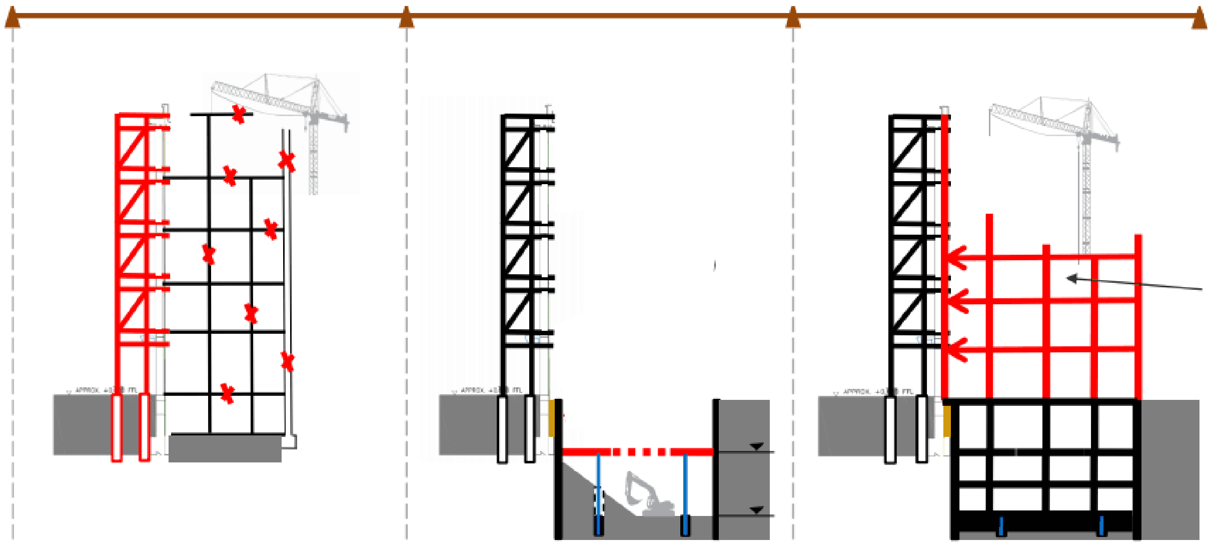


Fig. 1 Exterior view of the facade after the end of World War II in 1945

Fig. 2 Exterior view of the facade after reconstruction in 2016

The reason for the occurrence of vertical displacements was the process of reconstruction, which was started in 2013 and was carrying out in difficult geological conditions. This reconstruction project have two important features (see Fig. 3).



disassembling *parking construction* *new structures assembling*

Fig. 3 The stages of reconstruction

The first, during the reconstruction of the building, all structures were disassembled except for the facade one, which was kept by special metal retaining structures during reconstruction works. The second, with the height of the historical facade structure of 30 m, an underground parking of 20 m depth was built inside the building. The distance between the historical part of the facade that was kept by metal structures and the edge of the underground parking pit was only 5-6 m. Under such conditions, the vertical displacements in some places exceeded the allowable displacements by ten times and reached 110-120 mm for two years.
The results of geodetic monitoring for one year presented on Fig.4. On this figure presented the vertical displacements of deformation marks that were installed around the building facade. In order to better understanding of deformation process the vertical displacements presented in ascending order. From Fig. 4 is clear that deformation process is not uniform.

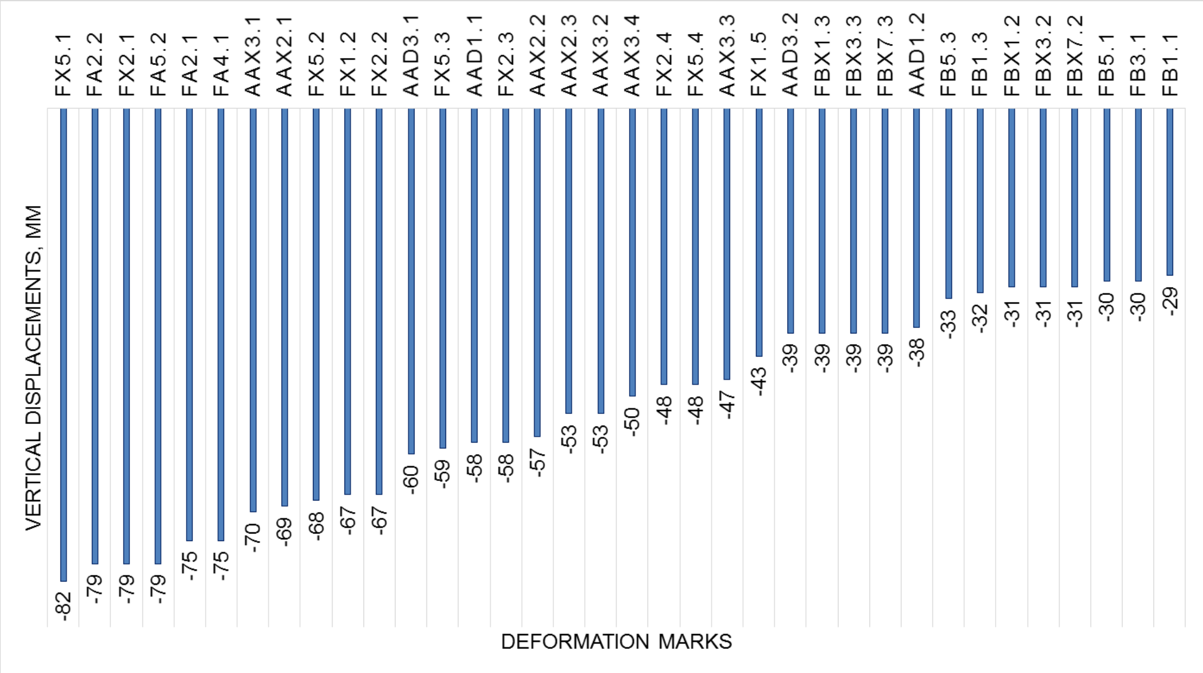


Fig.4 The vertical displacements at the end of the first year monitoring

These results were used for further researches.

2 PREDICTION BY USING RANDOM FUNCTIONS

In the study and prediction of structure vertical displacements, the most important should be considered two tasks:
- determination of the nature and distribution of vertical displacements within the structure;
- construction of adequate prediction model.

The first task is logically connected with the second. Determination of the nature and distribution of vertical displacements within the structure makes it possible to allocate the deformation blocks within which the deformation process is described by the same laws (equations). Thanks to this, it becomes possible within the block with statistically identical displacements do not build a prediction model for each individual deformation mark and to use the entire volume of measurements to construct a prediction model that will be describe the deformation process for any point within the block.

2.1 DEFORMATION BLOCKS DETERMINATION

Let's consider the first task of modeling displacements. Determination of the nature of the change in displacements within the structure is a very difficult task. As a rule, the separation of the structure into blocks with statistically identical displacements is not obvious. The situation is further complicated by the presence of gaps in observations, or change the time of observation cycles that happens at almost in monitoring object.

Based on the work (Gulyaev, Yu.P. et al., 2013), the authors presented their algorithm for allocation of homogeneous deformation blocks. As a criterion for the formation of statistically homogeneous blocks, the authors use the coefficient of variation $V_{\Delta S}$. In their works, they used next parameters: n - number of deformation marks; $\Delta\overline{S}$ - mean value of vertical displacements; $m_{\Delta S}$ - root mean square error (RMS) of vertical displacements; $V_{\Delta S}$ - coefficient of variation. At the first step the vertical displacements is placed in decreasing order. It is assumed that the mean values within the blocks are significantly different from the overall mean value, RMS within the block is minimal, and RMS within the blocks should be approximately equal. The vertical displacements group is considered homogeneous if $0,25 \le V_{\Delta S} \le 0,33$.

From our calculations the coefficient of variation equal 0.32 and therefore, the deformation process of whole facade is homogeneous. However, from Fig. 3 we can conclude opposite. Thus, despite the statistical validity, the proposed criterion for calculating the number of homogeneous deformation blocks does not always ensure adequate acceptable results. In addition, in this method there is some arbitrariness in choosing the coefficient of variation, which significantly reduces the accuracy of the approach.

We proposed to use an alternative method for calculating the number of deformation blocks. This method is based on the use of cluster analysis of vertical displacements. For the analysis, the horizontal coordinates of the deformation marks and the characteristics of the soils at their locations were used. The following methods were investigated: sum of distances, maximum distance and sum of squares of distances. In Fig. 5 present the results of a cluster analysis of vertical displacements by the method of sum of squares of distances.

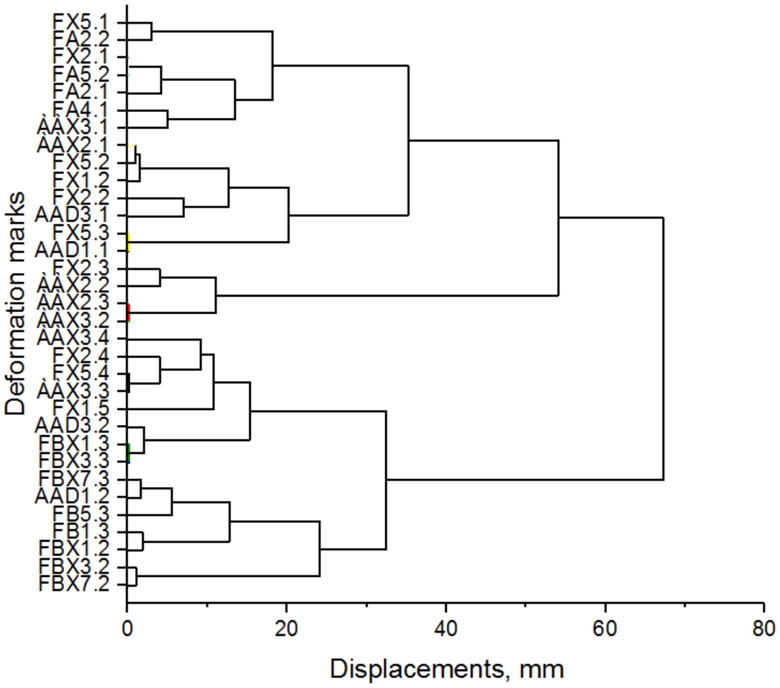


Fig. 5 Dendrogram of cluster analysis by the method of sum of squares of distances

All the results of the analysis, in contrast to the method of the coefficient of variation, showed the same distribution of all deformation marks in two blocks. Within each of these blocks, vertical displacements can be considered homogeneous. A necessary condition for the application of the theory of random functions is the normal distribution of the measured displacements in each block. Verification of displacements by the normal distribution was performed using the skewness and kurtosis criteria. Totally 113 cycles of observations were analyzed. By the results of this verification was determined that from 20^{th} cycle all displacements have a normal distribution. The minimum quantity of observations in the method of random functions equal to 5 (Gulyaev, Yu.P. et al., 2012). We used for future prediction models construction 93 cycles. Below, we will present and study the prediction models for the first block (displacements in the range from - 82 mm to - 50 mm).

2.2 PREDICTION MODEL CONSTRUCTION

The whole process of prediction model construction can be divided in four steps. First of all, we will give the short description of prediction model on basis of random functions.

2.2.1 Description of prediction model

The prediction kinematic model of the deformation process is constructed in the form of the following two functions (Gulyaev, Yu.P. et al., 2012):

$$\hat{m}_X\left(t_2/t_1\right)=\hat{m}_X\left(t_2\right)+\hat{r}_X\left(t_2,t_1\right)\frac{\hat{\sigma}_X\left(t_2\right)}{\sigma_X\left(t_1\right)}\dot{x}_i\left(t_1\right);\ \hat{\sigma}_X\left(t_2/t_1\right)=\hat{\sigma}_X\left(t_2\right)\sqrt{1-\hat{r}^2\left(t_2,t_1\right)}\quad(1)$$

where t_1 - the moment of the last observation cycle; t_2 - the moment on which the prediction of vertical displacement is performed; ^ - is a symbol of numerical parameters are obtained by means of prediction; $\hat{m}_{X_i}\left(t_2/t_1\right)$ - prediction of vertical displacement of deformation mark i at the moment t_2 under condition that we know: $\dot{x}_i\left(t_1\right)$, $\hat{m}_{X_i}\left(t_2\right)$, $\hat{r}_X\left(t_2,t_1\right)$, $\hat{\sigma}_X\left(t_2\right)$ - centered value of vertical displacement of deformation mark i at the moment t_2 and estimates of mathematical expectation, autocorrelation function and root RMS which are predicted at the time t_2; $\hat{\sigma}_X\left(t_2/t_1\right)$ - RMS that describes the expected error of prediction.

The construction of prediction kinematic model is reduced to the determination of statistical parameters of the distribution law of the process in each observation period and the subsequent approximation of these parameters in time. Stages approximation numerical estimates of parameters $\hat{m}_{X_i}\left(t_2\right)$, $\hat{r}_X\left(t_2,t_1\right)$, $\hat{\sigma}_X\left(t_2\right)$ discussed below.

2.2.2 Trend approximation $\hat{m}_{X_i}\left(t_2\right)$.

Using the results of measurements for the last 93 cycles, different models of trend prediction were investigated. The following types of models of different degrees were studied: polynomial (Kovačič, B. et al., 2009), power, exponential, piecewise linear, rational and logistic.

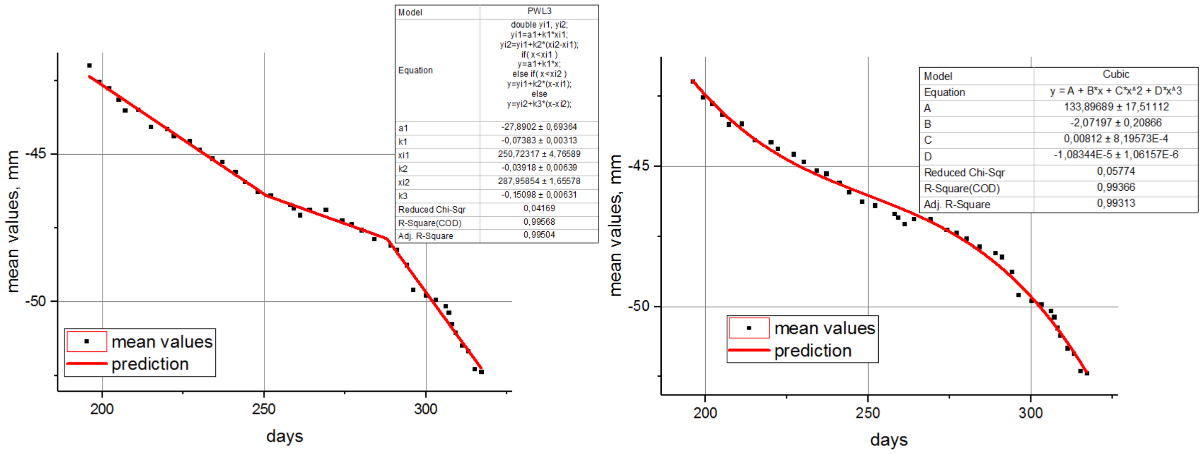


Fig. 6 Trend approximation by piecewise linear model of three segments (left) and third-degree (cubic) polynomial model (right)

Despite of the fact that many authors suggest using linear model, this type of model gave unsatisfactory results. For selection of the best model, the correlation coefficient, the accuracy of determining the model parameters, and the sum of the squares of the deviations were used. As a result, it was obtained that the best characteristics have piecewise linear model of three segments and the third-degree (cubic) polynomial model (see Fig. 6). These models were used for further research.

2.2.3 Root mean square error approximation $\hat{\sigma}_X(t_2)$

The search of the model parameters for RMS approximation was performed by the scheme similar to the search of trend parameters. As a result, it was obtained that the best characteristics have the piecewise linear model of three segments and the logistic model (see Fig. 7).

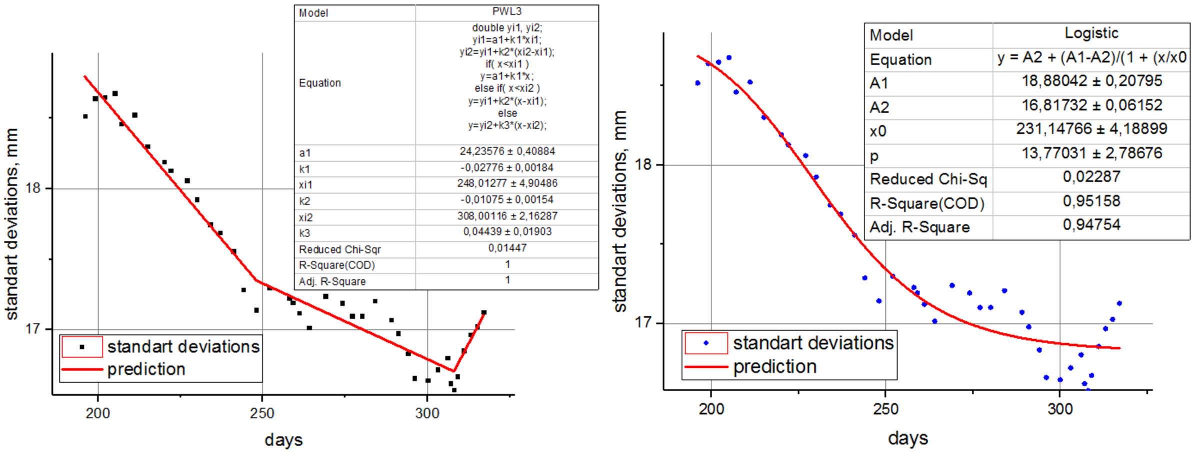


Fig. 7 RMS approximation by piecewise linear model of three segments (left) and logistic model (right)

These models were used for further research.

2.2.4 Computation and approximation of autocorrelation function $\hat{r}_X(t_2, t_1)$

The calculation of the autocorrelation function parameters of the deformation process is performed on the centered values of vertical displacements:

$$x_i(t_j) - m_X(t_j) = \dot{x}_i(t_j).$$

The correlation moments between the current measurements cycles of random process are calculated as:

$$K_x(t_{jk}, t_{jl}) = \frac{1}{n-1}\sum_{i=1}^{n}\left[\dot{x}_i(t_k)\dot{x}_i(t_l)\right], \tag{2}$$

where k, l - numbers of current measurements cycles; i - number of deformation mark.
The autocorrelation matrix is calculated as:

$$K_x = \frac{1}{n-1} S^T S, \tag{3}$$

where S – matrix of centered values of vertical displacements
Then, we turn to the normalized values:

$$r_x(t_{jk}, t_{jl}) = \frac{K_x(t_{jk}, t_{jl})}{\sigma_x(t_{jk})\sigma_x(t_{jl})}. \tag{4}$$

From the normalized autocorrelation matrix, calculate the correlation coefficients. For approximation of autocorrelation function, we used the same models as previously. As a result, it was obtained that the best characteristics have the third-degree (cubic) polynomial model and S-logistic model (see Fig. 8). These models were used for prediction models study.

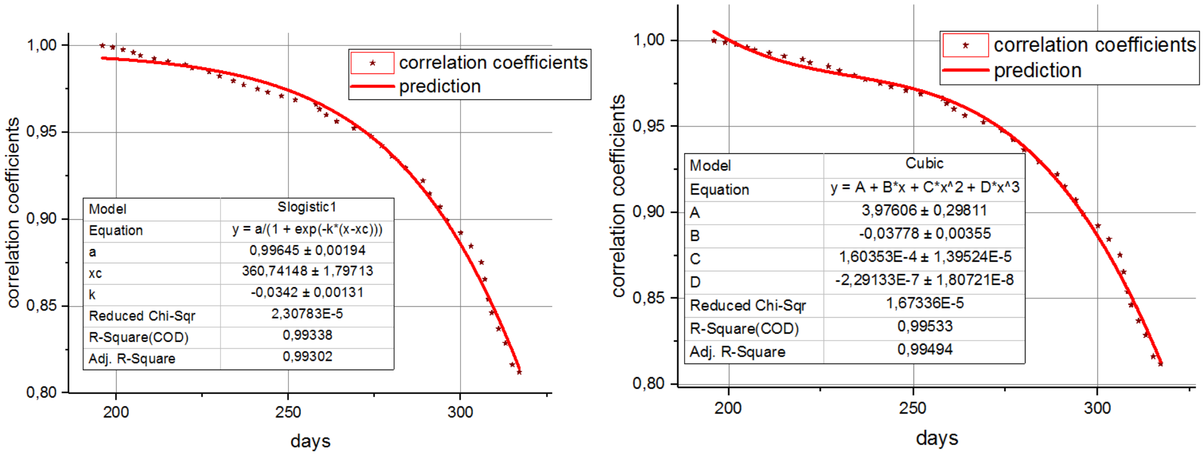


Fig. 8 Autocorrelation function approximation by S-logistic model (left) and third-degree (cubic) polynomial model (right)

3 STUDY OF PREDICTION MODELS

According to our previous research were formed twelve different prediction models (see Tab. 1). All of these models were investigated using equations (1).

Table 1. Prediction models

Model N	Trend function	RMS function	Autocorrelation function	Model N	Trend function	RMS function	Autocorrelation function
model 1	cubic	logistic	cubic	model 7	PWL3	logistic	cubic
model 2	cubic	logistic	Slogistic	model 8	PWL3	logistic	Slogistic
model 3	cubic	logistic	PWL3	model 9	PWL3	logistic	PWL3
model 4	cubic	PWL3	cubic	model 10	PWL3	PWL3	cubic
model 5	cubic	PWL3	Slogistic	model 11	PWL3	PWL3	Slogistic
model 6	cubic	PWL3	PWL3	model 12	PWL3	PWL3	PWL3

In order to define better model we made a prediction on ten cycles backward and compared prediction results with real displacements. By the deviations between predicted and measured values the RMS for each model was calculated. In such way we got the better model, which has a minimum RMS and which consists from the next equations (see model 2 in Tab. 1):

$$\hat{m}_X\left(t_j\right)=133.87-2.07t_j+0.0081t_j^2-0.000011t_j^3,$$

$$\hat{\sigma}_X\left(t_j\right)=16.82+\frac{2.06}{\left(1+\left(t_j/231.15\right)^{13.77}\right)},$$

$$\hat{r}_x\left(t_2,t_1\right)=\frac{0.996}{\left(1+e^{0.034\left(t_2-360.74\right)}\right)}.$$

These equations were used for the following forward prediction on ten cycles. One of the example of this prediction is presented in Fig.9.

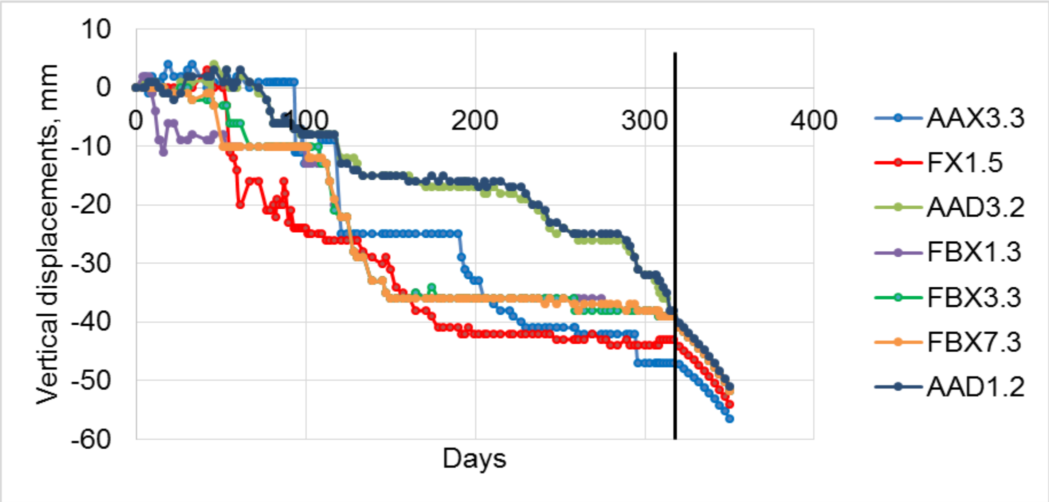


Fig. 9 Measured vertical displacements and prediction by the model 2 (after black line)

4 CONCLUSION

Summarizing the studies made, we can confidently state that the possibilities of the theory of random functions in the prediction of deformation processes have not been fully investigated yet. The results above is showing that an obligatory condition for using the theory of random functions is the obtaining of homogeneous deformation blocks. In solving this problem is well-established cluster analysis method. However, to solve this problem, ANOVA method can also be used which needs further investigation.

As a result of the study of various prediction models, optimal prediction models for the trend, RMS and autocorrelation function were obtained. The results of this study confirm that for complex deformation processes, simple linear and exponential models are ineffective. Further, to improve the quality of prediction models, it is recommended to conduct in-depth statistical analysis of measured vertical displacements in order to avoid blunders and systematic errors.

REFERENCES

Kovačič, B.; Kamnik, R.; Kapović, Z. 2009. Mathematical Analysis of Measured Displacements with Emphasis on Polynomial Interpolation, In *Geodetski list*: ISSN 0016-710X. Vol.63 (86), No. 4 (2009), pp. 315-327.

Gulyaev, Yu.P., Khoroshilov, V.S. 2012. Mathematical modeling. Analysis and prediction of deformations of structures based on geodetic data based on the kinematic model: lecture notes, Engineering Geodesy, SSGA, Novosibirsk, 93 p.

Gulyaev, Yu.P., Khoroshilov, VS, Pavlovskaya, O.G. 2013. Mathematical modeling. Identification of hidden laws in the dynamics of landslide slopes in conditions of explosive influences and removal of large masses of soil according to geodetic data: lecture notes, Geodesy, SSGA, Novosibirsk, 131 p.