Precise Local Geoid Determination to Make GPS Technique More Effective in Practical Applications of Geodesy

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Key words: Geoid, GPS/Levelling, Surface Fitting, Collocation, Height

SUMMARY

Global Positioning System (GPS) has the benefits of high accuracy and simultaneous 3-D positioning in Geodetic aims, however, GPS derived ellipsoidal heights must be transformed to orthometric heights to have any physical meaning in a surveying or engineering applications.

Obtaining orthometric heights as related to GPS measurements is one of the most important research areas of Geodesy. The motivation for such research is to obtain the orthometric heights with decreased amount of spirit levelling measurement work, which is the conventional way of determining these heights. The main advantage of levelling measurements is the high accuracy. On the other hand, levelling measurements require high cost and lengthy observation time and with these aspects, it is not convenient for the practical geodetic purposes. So, one another way of obtaining orthometric heights from GPS measurements is to have a precise geoid model in working area. Actually, a precise geoid model is very important part of a complete and reliable geodetic infrastructure. In this paper, the importance of having a precise geoid model in practical applications of geodesy, while GPS technique is being involved so much in to the applications will be emphasized. Especially the necessity of local geoid models in a national geodetic infrastructure will be indicated and precise determination of a local geoid model using GPS and Levelling data will be summarized with its methodology and results as a case study.

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1. INTRODUCTION AND BACKGROUND

Determination of geoid has been one of the main research areas in Science of Geodesy for decades. According to wide spread use of GPS in geodetic applications, great attention is paid to the precise determination of local/regional geoid with an aim to replace the geometric levelling, which is very onerous measurement work, with GPS surveys.

GPS technique provides the surveyor with three-dimensional coordinates including ellipsoidal heights (h) with respect to its reference ellipsoid, the geocentric WGS84 (World Geodetic System 1984). As in GPS measurements, geodesists have chosen an oblate ellipsoid of revolution, flattened at the poles, to approximate the geoid in order to simplify survey data reduction and mapping. However, most surveying measurements are made in relation to the geoid, which is the equipotential surface of the earth gravity field, not ellipsoid because the equipment is aligned with the local gravity vector, which is perpendicular to the geoid surface, usually through the use of a spirit bubble (Featherstone 1998).

Because of these facts, ellipsoidal heights can’t satisfy the aims in practical surveying, engineering or geophysical applications as they have no physical meaning and must be transformed to orthometric heights (H), which are referred to geoid, to serve the geodetic and surveying applications. To accomplish this transformation between the ellipsoidal heights and orthometric heights is just possible to know the undulation of geoid from the ellipsoid (N). Basically a WGS84 ellipsoidal height (h) is transformed to an orthometric height (H) by subtracting the geoid-WGS84-ellipsoid separation (N), which is called as geoid undulation.

Depending on data availability and accuracy requirements, there are two principle approaches for determining geoid models, which utilizes to transform GPS ellipsoidal heights to orthometric heights. These approaches include gravimetric method and interpolation between geometrically derived geoid heights using the benchmarks of which three dimensional coordinates and orthometric heights have been determined according to GPS and levelling measurements.

In this paper, precise local geoid determination will be considered according to geometric method using GPS/Levelling data. First of all, an overview of general methodologies which are applied for geoid determination in local areas will be given. Then, a case study and the results of it will be summarized. The study have been carried out using the GPS and Levelling data in an approximately 50x60 km^2 area and the density of benchmarks that were used is approximately 3 points/km^2. The topography of the area is not uniform and extremely rough (see figure 2 for the topography of the region). During the modeling of the geoid in the study area, Least Squares Collocation method has been used. The theory of the technique will be explained. Also, the importance and necessity of local geoid models as the part of geodetic infrastructure of a country will be emphasized with Turkey example in this paper.

2. GEOID MODELING FROM A GENERAL PERSPECTIVE

It is possible to group geoid models a global, regional and local with respect to covered area of the model. Determining each of these models has composed one of the main interests of Geodesists. More and more accurate geo-potential models (earth models) have been developed. The modern models can provide the geoid heights of any points on the earth surface with an accuracy ranging from 30 cm to a few meters (Rapp 1997; Chen and Yang 2001).

By the extensive use of GPS technique with geodetic aims, great interest has been collimated to the precise determination of local/regional geoid with an aim to replace levelling measurements with GPS surveys. There are two basic approaches as being the geometric approach and the gravimetric approach (Featherstone 1998; Chen and Yang 2001). In a relatively small area, the local geoid can be determined by a combination of GPS derived heights and levelling heights. This approach is called as Geometric approach. From GPS derived heights (h) and levelling heights (H) at some points called as benchmarks (reference points), geoid heights (N) (geoid undulations) are calculated according to basic relation, "N = h-H". The geoid heights at any other GPS measurement points can be interpolated analytically or graphically as being upon to the known geoid heights of reference points. This procedure can be carried out by *transformation*, *pointwise interpolation* or *surface interpolation* with determining the parameters of the surface and by this way describing the geoid as an analytical surface.

A plane or low order polynomial is usually used to model the geoid surface (Featherstone et al. 1998). The geometric method has been mostly preferred for the model used in practical applications of geodesy, i.e. for large scale map production, engineering projects etc.

There are several factors that affect the accuracy of geoid model determined according to geometric approach. These are;

- Distribution and number of reference stations (GPS/Levelling Stations): the distribution of these points must be as homogeneous as possible on the model area and these points must be chosen to figure out probable changes of geoid surface. For this reason, to consider the topographic properties while choosing these reference points might be beneficial. On the other hand, in the Regulation for Producing Large Scale Maps and Geospatial Data in Turkey, the number of reference points is indicated as at least 1 point/20km^2 for local geoid model determination (see. Anonym 2003).
- The accuracy of GPS derived ellipsoidal heights (at least ±3.5 cm in Anonym 2003) and the accuracy of levelling heights (H) (at least ±5 mm/km in Anonym 2003)
- Characteristic of the geoid surface
- Modeling method: it is impossible to say that a unique method works properly for local geoid models in different areas. So, it has to be given an effort to determine the most appropriate geoid model for a local area by trying different modeling methods.

The gravimetric approach is to determine a geoid using gravity measurements. In the determination of a local and regional gravimetric geoid the remove-restore technique is widely used (see. Featherstone et al. 1998, Yang and Chen 1999, Yang and Chen 2000, Chen and Yang 2001). However, because of that the major motivation of this study is geometric approach; it is not going to be given the details on remove-restore technique (for more details it can be seen Anonym 2002).

3. SURFACE FITTING METHODS FOR GEOID MODELING

Among the geoid modeling techniques, fitting a surface, which is based on the reference points that have been chosen in the most critical locations for representing of the geoid, is one of the most common method. Representing geoid heights as an analytical surface and deriving the geoid undulation values in new points, which are measured with GPS technique, according to the mathematical formulation of this surface constitutes the basic idea of this technique. However, in a local area, determined geoid model with surface fitting just works in the coverage area of the reference points properly. The model doesn't give reliable results for the extrapolation points. Researches shows that this method gives better results where the geoid have a regular trend and with well distributed and intensive reference points.

There are different kinds of interpolation techniques used for modeling the geoid heights. In some part of these techniques, the a-priori heights derived from measurements assumed as if they are exact values, in another part of the techniques, an adjustment procedure is carried out and random errors are filtered before using the mathematical expressions for interpolation.

Mainly the interpolation is handled according to three approaches. These are pointwise (i.e. interpolation with weighted average), simultaneously with a function (i.e. interpolation with polynomials), patchwise (i.e. interpolation with summation of surfaces) (Yanalak 2001).

In this study, which is presenting as a case study in here, the geoid model of the working area was expressed with fifth degree polynomial. This polynomial constitutes the trend function (deterministic part) of a collocation problem. The geoid undulations of interpolation points were determined according to Least Squares Collocation (LSC) method. Collocation is the most general form of the adjustment process which includes least squares adjustment, filtering and prediction (interpolation, extrapolation) steps with in a combined algorithm (Şerbetçi and Öztürk 1992; Yiğit 2003). With another saying, the model that includes a second random variable (signal) in the mathematical model of it than the residuals (noises) as its differences from the known least squares adjustment procedure is called as collocation (Yiğit 2003).

Collocation method comprises several steps (Şerbetçi and Ergun 1992, Yiğit 2003). These steps are:

- Determining the measurement errors (noise, $-\underline{v} = \underline{n}$) and signals ($\underline{s}$) separately (*filtering*) or deriving adjusted measurements,
- Computing the unknowns of model function (*adjustment*),

- Determining the $\underline{s}_P$ signals and λ_P interpolation values in the new point of which the parameters will be derived (*prediction*),
- Computing root mean square errors of measurements and derived parameters.

General Collocation model is (Şerbetçi and Ergun 1992, Ayan et al 1996b):

$$\underbrace{\underline{l} + \underline{v} = \underline{A}\,\underline{x}}_{\text{Least Squares Adjustment}} + \underbrace{\underline{R}\,\underline{s}}_{\text{Signals}} \qquad (1)$$

$\underline{l}$ and $\underline{v}$	:	vectors of observations and their residuals
$\underline{x}$	:	unknown parameters vector
A	:	coefficients matrix of adjustment
$\underline{s}$	:	signals in reference points
$\underline{R}$	:	linearized correlations between observations and signals

In the simple prediction problems, the model is:

$$\underline{l} = \underline{s} \qquad \textit{Signals in reference points} \qquad (2)$$

$$\underline{P} = \underline{Q}^{-1} \qquad \textit{Weight matrix (Stochastic model)} \qquad (3)$$

$$\underline{v}^T \underline{P}\,\underline{v} = min \qquad \textit{Condition of Least Squares Adjustment} \qquad (4)$$

$$\underline{D} = \underline{C}_{vv} + \underline{C}_{ss} \qquad (5)$$

$$\underline{f} = s' = \underline{C}_{s's}\,\underline{D}^{-1}\,\underline{l} \qquad \textit{Signals in Interpolation Points} \qquad (6)$$

4. NUMERICAL EXAMPLE

As it is mentioned in previous titles, geoid models are also grouped as global, regional and local and the precision of the model increases respectively. As it is mentioned, GPS/Leveling method is appropriate for estimating geoid heights and establishing a geoid model with centimeter accuracy in a limited area. Expressing the interpolated heights with least squares collocation method is one of the alternative computation methods. This method was used in this study and the details and also results of the mentioned study are going to be given in following titles as the case study. But, before explaining the case study on local geoid modeling with least squares collocation method, it is thought to be better to give some information about geoid modeling studies in Turkey.

4.1 Geoid Modeling in Turkey

So far, several national geoid models have been determined for Turkey as a part of geodetic infrastructure studies. Detailed information can be provided from Ayhan et al 2001 about the ex-models. The latest one is "Turkey Geoid 1999A" (Ayhan et al 2002). This regional model of Turkey geoid is gravimetric geoid, which has been improved using GPS/Levelling data, Satellite Altimetry data, hydrographic measurements data and also using Digital Terrain models. However, the accuracy of TG99A is not the same for each part of the country (see Ayhan et al 2002). In some part of the area, this national regional geoid model can not satisfy the accuracy, which is necessary for most of the routine geodetic applications. There are some regions that the accuracy of the model is in meters (see Çelik et al 2002). So, the necessity of studying on local geoid model appeared because of the national geoid model as a solution method. As it is well known, using more intensive and homogeneous distributed data and concerning the changes of geoid surface with in this limited area while modeling the geoid results increased accuracy of the model. With these aims, there are several local geoid modeling studies were carried out. The locations of these study areas are indicated on map (see figure 1).

Figure 1: The regions where local geoid modeling studies were carried out in Turkey. The squares give an idea about the relative sizes of the coverage areas of the geoid models (Anonym 2001, Erol and Çelik 2003). Izmir is pointed out with an arrow

4.2 İzmir Local Geoid Model

Izmir is located in the west of Turkey just beside of the Aegean Sea (see figure 1, in the map, the location of Izmir is indicated by an arrow). The area of the study covers 50x45 km^2 area of this region. For modeling the geoid of this area, a fifth degree polynomial was used as a trend function (see equation 8). And by this way, geoid undulations were being expressed as depending on geodetic latitude (φ) and geodetic longitude (λ). In the equation, these coordinates of reference points were normalized to use (see equation 9).

$$N(X,Y) = \sum_{k=0}^{5} \sum_{\substack{j=k-i \\ i=0}}^{k} a_{ij} y^i x^j \qquad \textit{general notation} \qquad (7)$$

$$N(X,Y) = A_0 + A_{01}X + A_{10}Y + A_{02}X^2 + A_{11}XY + A_{20}Y^2 +$$
$$A_{03}X^3 + A_{12}X^2Y + A_{21}XY^2 + A_{30}Y^3 + \quad (8)$$
$$A_{04}X^4 + A_{13}X^3Y + A_{22}X^2Y^2 + A_{31}XY^3 + A_{40}Y^4$$
$$A_{05}X^5 + A_{14}X^4Y + A_{23}X^3Y^2 + A_{32}X^2Y^3 + A_{41}XY^4 + A_{50}Y^5$$

$$X = k*(\varphi - \varphi_0) \;\; Y = k*(\lambda - \lambda_0) \;\; k = 100/\rho^0 \quad (9)$$

$\varphi_0 = 38^0.4090196$ $\lambda_0 = 27^0.1013351$ *as geometric center of model area* (10)

The model was accomplished using 302 benchmarks in total without any inconsistent point, because during the modeling studies probable inconsistent points have been removed. 181 of these benchmarks were taken as modeling pins, and the rest 121 points were chosen for testing the model. While choosing these test points, the homogenous distribution and topogragraphic properties are considered (the distributions of both modeling points and test points can be seen in figure 2). The geographic coordinates including heights (h) are known in ITRF96 datum and also the practical heights (H) are known from geometric levelling in the datum of Turkey National Vertical Network (TUDKA). There is a detail that is necessary to mention in this point: practical heights of benchmarks are used instead of their orthometric heights in Turkey. That is why the orthometric heights are derived by adding gravimetric corrections to the levelling measurements naturally. The orthometric heights of the points of Turkey National Vertical Datum have been derived by this way, but, on the other hand the heights of new points derived from these triangulation points are calculated with out gravimetric corrections. This situation doesn’t couse any problem related to accuracy in routine geodetic and surveying applications such as large scale map production, GIS applications etc. (Ayan et al 2001, Erol and Çelik 2003).

For modeling the geoid of the region, geoid heights in the benchmarks were calculated according to equation 11 to use the coefficients of the fifth degree polynomial according to least squares adjustment method.

$$N = h_{ITRF96} - H_{Nat.Ver.Datum} \quad (11)$$

In the equation, $H_{Nat.Ver.Dat.}$ is practical height in National Vertical Datum, h_{ITRF96} is ellipsoidal height that is referenced to GRS80 ellipsoid and derived from GPS measurements and N is geoid height of corresponding benchmark (Erol and Çelik 2003).

After computing the 21 unknown parameters of the fifth degree polynomial, four of these coefficients were found as not significant. These were the coefficients of “X^4”, “X^2Y^2”, “Y^4” and “XY^4” terms. These terms were removed from the polynomial and the rest of the parameters were computed again. By the way, the number of coefficients was reduced to 17.

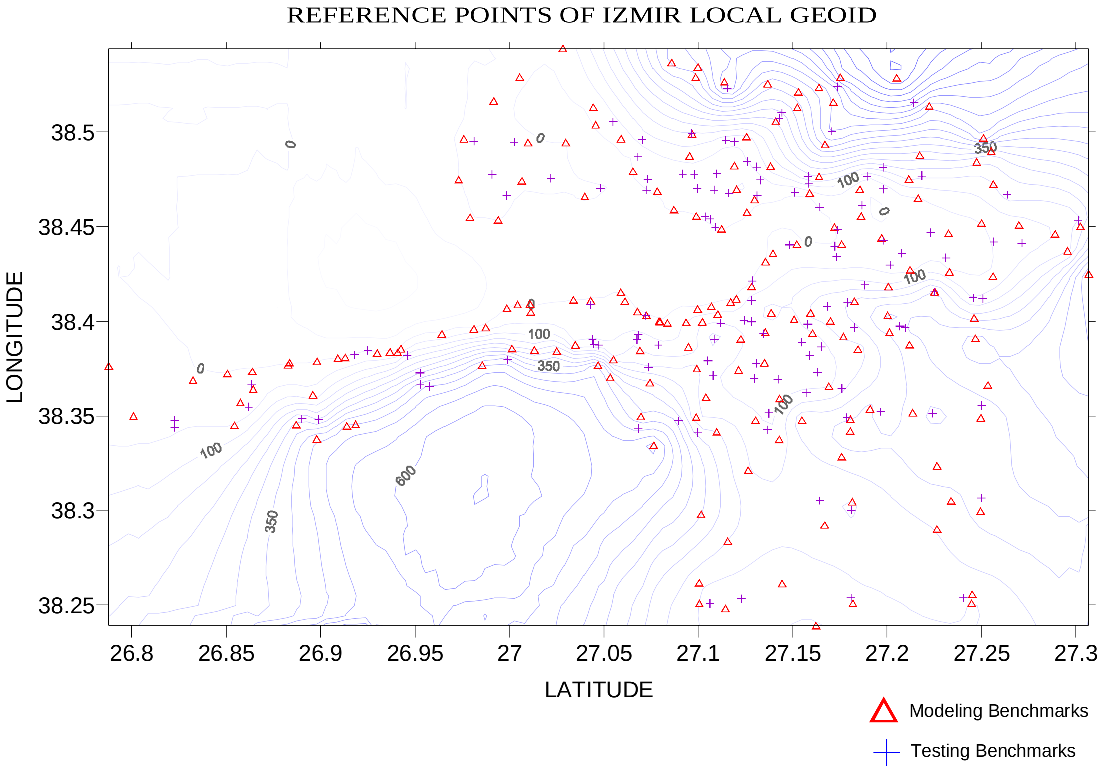

Figure 2: In the figure, it can be seen that the distributions of modeling benchmarks and test benchmarks in the area of study

After generating the polynomial as the deterministic part of the computation algorithm, a simple collocation method was applied. The procedure is for computing the signals of both modeling points and test points is as given below.

The general procedure of a simple least squares collocation method has been given in the equations between 1 and 6. In this model as the second part of the algorithm apart from determining the coefficients of the polynomial;

$$\underline{l} = \underline{v}_N = \underline{s} \tag{12}$$

$$\underline{s}' = \underline{C}_{ss'}\, \underline{D}^{-1} \underline{v}_N \tag{13}$$

$$\underline{D} = \underline{I} + \underline{C}_{ss} \tag{14}$$

The notation has been explained in the previous title, so there isn't necessity to say it again in here. To express correlations between the v_N values in reference points, a covariance function has to be used. In this study, Hirvonen function (see equation 15) was chosen with this aims as considering the previous studies carried out for geoid modeling with LSC method.

$$C_{s_i s_j} = \frac{1}{1 + \left(\frac{S_{ij}}{c}\right)^2} \qquad \textit{Hirvonen Function} \tag{15}$$

In Hirvonen function, c is an ampric value. C_{sisj} is the correlation between v_{Ni} and v_{Nj} values (residuals of geoid undulations) of for example P_i and P_j points. S is the distance in km between these two points. On the other hand, the elements of $\underline{C}_{ss'}$ matrix are the expression of the correlation between reference point and interpolation point. $\underline{I}$ is a unit matrix. In this study, c coefficient is choosen as 7 by trying. After calculating the signals $\underline{s}'$, these were added to geoid undulations, which were derived from polynomial for the 121 test points. By this way, the resultant geoid undulations were been calculated according to geoid model determined using LSC method.

$$\underline{N}_{LSC\_Model} = \underline{N}_{Polynomial} + \underline{s}'$$

4.2.1 Testing of the Model

For testing the created geoid model, 121 test points of which geoid heights had already been known from the GPS and levelling measurements, were used. After collocation method, the root means square error, computed according to real errors (ε), has been found *±3.9 cm*. This value also expresses the fitting level of the model to the data. The maximum error was found 10.41 cm. And the number of points whose error is between ±5 cm interval is 18 among the 121 points. It is also seen that the maximum errors were calculated for the exterior points of the model area and this situation is natural (as it can be seen in figure 3).

5. CONCLUSION AND FUTURE PLANS

The major motivation of this study was to emphasize what a precise geoid model means for routine applications of geodesy and surveying, which are carried out using GPS positioning technique. Large scale map production for different purposes, GIS applications, engineering surveys etc can be said in comprises of these applications. Obtaining physical heights of points from GPS measurements is a very important handicap in application. However, it is implied that to overcome this problem via having a precise geoid model in the region of study is the most advantages solution. On the other hand, applying levelling measurements to obtain physical heights of new points in each time instead of deriving them from just GPS measurements is not practical at all. Because, as is very well known, levelling measurements needs high cost and lengthy observation time although that provides high accuracy in height.

Utilizing a precise geoid model, orthometric heights or practical heights (as in the case study of this paper) are obtained from GPS measurements. Because of that, the studies of geoid modeling have increased its importance after widespread use of GPS technology in Geodetic aims. Also geoid modeling has become indispensable part of geodetic infrastructure studies.

From this view point, there several methods for modeling the geoid and available data and required accuracy of the model are two essential criteria while deciding "which method will be used". Investigations were shown that the required accuracy level of the model can be reached by employing geometric approach in local studying area using GPS and Levelling data (see Ayan et al 1996a-b, Ayan et al 2001, Chen and Yang 2001, Çelik et al 2002, Zhicai and Chen 2002, Anonym 2003, Erol and Çelik 2003). However, to be possible reach higher accuracies with combining more data groups such as gravity, satellite altimetry and geopotantial model in a complete algorithm in regional manner, for practical geodetic works, geometrical solution was found satisfying enough (Erol and Çelik 2003) (see Featherstone et al 1998).

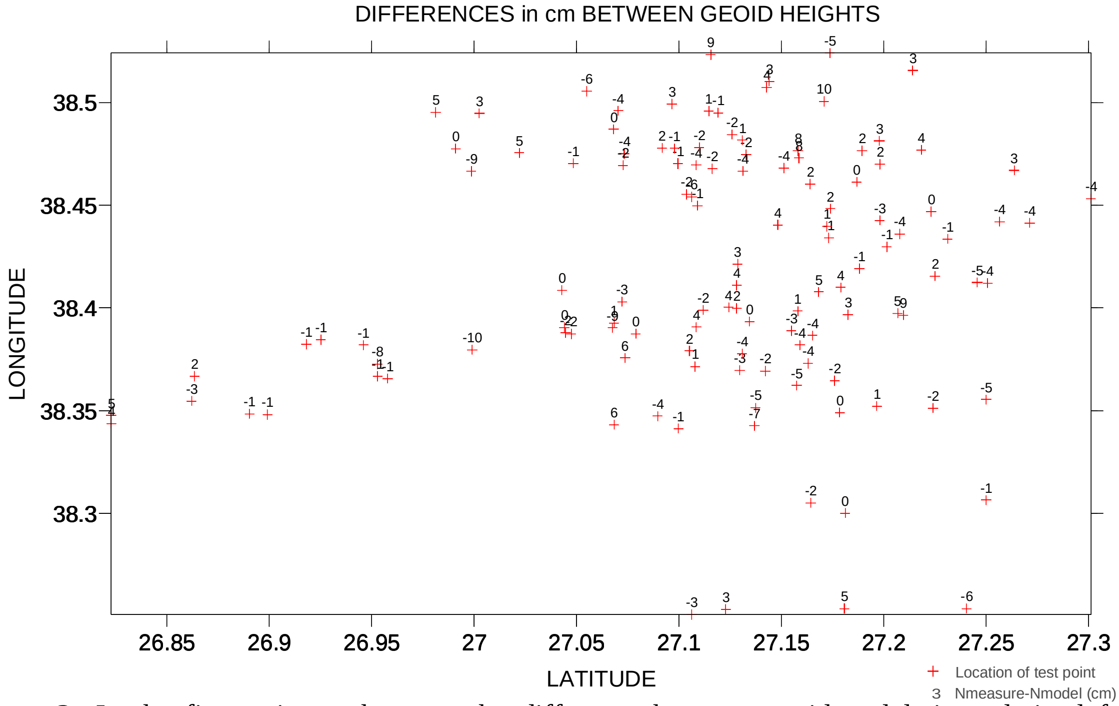

Figure 3: In the figure, it can be seen the difference between geoid undulations derived from GPS/levelling measurements and the ones computed according to model.

Modeling geoid using multiparameter high degree polynomial as depending on well distributed and sufficient number of GPS/Levelling points that are called as benchmarks and by the way expressing the surface of geoid as an analytical surface is one of the ways of modeling. Thereafter, it is possible to purify this polynomial that means to make it more accurate with some techniques is possible. Applying weighted corrections from the reference points to the interpolation points or according to least squares collocation method can be considered in to these techniques. As an example, the case study was handled in here. In this numerical investigation, local geoid model of 50x60 km^2 of Izmir metropolitan region was computed using least squares method. The trend function of the model was decided as fifth degree polynomial by trying. At the end of the modeling study, the applicable accuracy of the model was calculated ±3.9 cm for interpolation points. And this accuracy is sufficient for practical geodetic applications as depending on Regulation for Producing Large Scale Maps and Geospatial Data in Turkey (Anonym 2003). According to personal opinion, it will be possible to increase this accuracy level by trying other modeling techniques. So, in this manner, in the future studies, other techniques will be tried using different test points' variations. Such as, this time, 121 test points were used to test the model and it might be possible to obtain better results by collocation method by reducing the number of test points and increasing the number of model points.

There is another point is that there is necessity to develop a solution for local geoid model, which works properly for interpolation points and also for extrapolation points. This will be possible to combine a local geoid model with Turkey National geoid model, TG99A. Of course this kind of model will be work with a better accuracy for interpolation points, however, there might be possible to obtain an acceptable accuracy level for extrapolation points in practical applications. This is also included in the future plans.

While it is coming at the end of paper, this is necessary to say one more time that local geoid models with sufficient accuracy are and will be important for the geodetic infrastructure in vertical in Turkey, at least until national geoid model of Turkey will reach the applicable accuracy level in practical geodetic applications.

REFERENCES

Aksoy, A., Ayan, T., Çelik, R. N., Demirel, H., Deniz, R., Gürkan, O. (1999) "Geodetic Infrastructure for Geographic Information Systems Under the Enlightening of Recent Developments and Exposed Problems", Union of Chambers of Turkish Engineers and Architects-Chamber of Surveying Engineers, Education Series, Istanbul, Turkey, (*in Turkish*) pp. 1-25

Anonym (2001) "Encarta World Atlas 2001", Electronic Atlas on Compact Disc, 2001 Version

Anonym (2002) "International School for Determination and Use of the Geoid", Volume 1, Lecture Notes, Publication of International Geoid Service (IGeS), DIIAR-Politecnico di Milano

Anonym (2003) "Regulation for Producing Large Scale Maps and Geospatial Data in Turkey", Ankara, Turkey (*in Turkish*)

Ayan, T., Çelik, R. N., Alanko, M. G. (1996a) "Densification and Refining of Izmir Metropolitan Triangulation Network using GPS Technique Project", Technical Report, Istanbul Technical University-Civil Engineering Faculty, Istanbul, Turkey (*in Turkish*), 89-94

Ayan, T., Çelik, R. N., Alanko, M. G. (1996b) "Densification and Refining of Izmir Metropolitan Triangulation Network using GPS Technique Project 20/09/1996 dated Additional Report", Technical Report, Istanbul Technical University-Civil Engineering Faculty, Istanbul, Turkey, (*in Turkish*), 1-15

Ayan, T., Baykal, O., Aksoy, A., Deniz, R., Baz, İ., Arslan, E., Şahin, M., Coşkun, Z., İnce, C., D., Üstüntaş, T. (1996c) "Zonguldak GPS Network Project", Technical Report, Istanbul Technical University-Civil Engineering Faculty, Istanbul, Turkey, (*in Turkish*), 49-64

Ayan, T., Deniz, R., Çelik, R. N., Denli, H. H., Özlüdemir, M. T., Erol, S., Özöner, B. (1999) "İstanbul GPS Triangulation Network (İGNA) Project", Technical Report, Istanbul Technical University-Civil Engineering Faculty, Istanbul, Turkey, (*in Turkish*), 42-51

Ayan, T., Deniz, R., Çelik, R. N., Denli, H. H., Özlüdemir, M. T., Erol, S., Özöner, B., Akyımaz, O., Güney, C. (2001) "İzmir Geodetic Reference System-2001 (IzGRS-2001) Project", Technical Report, Istanbul Technical University-Civil Engineering Faculty, 2000/2294, Istanbul, Turkey, (*in Turkish*), 44-57

Ayhan, M., E., Lenk, O., Demir, C., Kılıçoğlu, A., Kahveci, M., Türkezer, A., Ocak, M., Açıkgöz, M., Yıldırım, A., Aktuğ, B., Şengün, S., Kurt, İ., Fırat, O. (2001) "Turkey National GPS Network 1999 (TUTGA99)-Technical Report", Technical Report, General Command of Mapping, Geodesy Department, Ankara, Turkey, (*in Turkish*), 26-34

Ayhan, M. E., Demir, C., Lenk, O., Kılıçoğlu, A., Aktuğ, B., Açıkgöz, M., Fırat, O., Şengün, Y. S., Cingöz, A., Gürdal, M. A., Kurt, A. İ., Ocak, M., Türkezer, A., Yıldız, H., Bayazıt, N., Ata, M., Çağlar, Y., Özerkan, A. (2002) "Turkey National GPS Network -1999A (TUTGA-99A)", ISSN 1300-5790, Turkey Republic General Command of Mapping, Special Edition 16, General Command of Mapping Press, Ankara, Turkey, (*in Turkish*), 47-54

Chen, Y. and Yang, Z. (2001) "A Hybrid Method to Determine the Hong Kong Geoid", International Conference, FIG Working Week 2001, Seoul, Korea, 6-11 May 2001

Çelik, R., N., Ayan, T., Erol, B. (2002) "Determining Geoid Model for Geodetic Infrastructure of Marmara Earthquake Region Land Information System (MERLIS) Project Technical Report", Project by Assoc. Prof. Rahmi N. Çelik (ITU 2002/06/209 Project Number) (*in Turkish*)

*Erol, B., Çelik, R. N., Erol, S. (2003***)** "Using GPS and Leveling Data in Local Precise Geoid Determination and Case Study", Abstract, Geophysical Research Abstracts, Vol. 5

Erol, B., Çelik, R. N. (2003) "Investigation on Local Precise Geoid Determination Using GPS and Levelling Data", "International Symposium of Modern Technologies, Education and Professional Practice in Globalizing World", 6-8 November 2003, Sofia, Bulgaria

Featherstone, W. E., Denith, M. S. and Kirby J. F. (1998) "Strategies for the accurate determination of Orthometric Heights", Survey Review, Vol 34, No 267, January 1998, pp.278-296

Lee, J. T. and Mazera, D. F. (2000) "Concerns Related to GPS-Derived Geoid Determination", Survey Review, Vol. 35(276), pp.379-397

Rapp, R. H. (1997) "Global Models for 1-cm Geoid-Present Status and Near Term Prospects", Geodetic Boundary Value Problems in View of one centimeter geoid (eds. Sanso, F. and Rummel, R.), Springer Publishing., ISB N 3-540-62636-0, pp.273-308

Şerbetçi, M., Öztürk, E. (1992) "Adjustment III", Karadeniz Technical University, Engineering and Architecture Faculty, General Publication Number 144, Faculty Publication Number 40, Trabzon, Turkey (*in Turkish*)

Yanalak, M., Baykal O. (2001) "Transformation of Ellipsoid Heights to Local Leveling Heights", Journal of Surveying Engineering, Vol. 127, No.3, August-2001, ASCE, Paper No.21474, 90-103

Watson, D., F. (1992) "Contouring: A Guide to The Analysis and Display of Spatial Data", Pergamon Press, ISBN 0-08-040286-0, Volume 10, First Edition 1992

Yang, Z. J. and Chen Y. Q. (1999) "Determination of Local Geoid with Gravimetric Method: Case Study", Journal of Surveying Engineering, Vol.125, No.3, ASCE, pp.136-146

Yiğit, C. Ö. (2003) "The Comparison of the Interpolation Methods Used in Transformation of Ellipsoidal Heights to Orthometric Heights", Selcuk University, Graduate School of Natural and Applied Sciences, Department of Geodesy and Photogrammetry, Master Thesis, Konya, Turkey (*in Turkish*)

Zhicai, L., Chen, Y. Q. (2002) “Precise Determination of Hong Kong Geoid Using Heterogeneous Data”, April 19-26 2002, FIG XXII International Congress, Washington D.C., USA

BIOGRAPHICAL NOTES

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