On the Application of Nonparametric Regression Methods to Geodetic Data

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Key words: Collocation and filtering, nonparametric regression, triangulation of coordinate transformation, motion of sliding slopes

SUMMARY

The collocation and filtering method of geodesist H. Moritz not only estimates parameters but also separates signal from noise and provides prediction following the Wiener-Kolmogorov theory and treating the signal as a stochastic quantity. The statistic approach of nonparametric or semiparametric regression simplifies the theory of the method. The signal is treated as a deterministic quantity, and the Wiener-Kolmogorov prediction can be replaced by simple interpolation. In addition, several methods to determine the signal to noise ratio such as cross validation were developed. By applying these methods to the transformation of old coordinates from 1903 to new GPS-coordinates of 1995, systematic distortions of the old coordinates can be estimated besides the estimation of Helmert-transformation parameters. Mapping the derivative of the systematic distortions enabled the Swiss Canton of Basel-Stadt to choose appropriate triangles in order to define a triangulation for coordinate transformation which fulfilled precision requirements in each triangle separately. These statistical methods may also be applied to the estimation of locally different motions of sliding slopes.

ZUSAMMENFASSUNG

Mit der Kollokations- und Filterungsmethode des Geodäten H. Moritz werden neben der Schätzung von Parametern Rauschen und Signal voneinander getrennt. Mit der Wiener-Kolmogorov-Prädiktion kann das als stochastische Grösse aufgefasste Signal an Neupunkten geschätzt werden. Der statistische Ansatz der semiparametrischen Regression vereinfacht die Theorie. Das Signal wird als deterministische Grösse behandelt, und an die Stelle der Wiener-Kolmogorov-Prädiktion tritt die Interpolation der geschätzten Signalfunktion. Das Signal zu Rausch Verhältnis wird durch ein Regularisierungsverfahren, z.B. Kreuzvalidierung, festgelegt. Mit diesem Ansatz können bei der Umrechnung der alten Landeskoordinaten von 1903 in die neuen GPS-basierten Landeskoordinaten von 1995 die Parameter der Helmerttransformation und die systematischen Verzerrungen der alten gegenüber den neuen Koordinaten geschätzt werden. Die grafische Darstellung der Ableitung der systematischen Verzerrungen als Fläche im Raum ermöglichte es, im Kanton Basel-Stadt (Schweiz) eine Triangulation derart festzulegen, dass bei der dreiecksweisen Umrechnung zwischen den beiden Koordinatensystemen die vorgegebenen Genauigkeiten eingehalten werden. Mit dem semiparametrischen Ansatz kann auch das lokal unterschiedliche Rutschen eines Rutschhanges dargestellt werden.

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1. INTRODUCTION

In Switzerland there are two sets of national coordinates. The conventionally measured ones from 1903 we denote by LV03 ("Landesvermessung 1903"), and the GPS-based set of 1995 is called LV95 ("Landesvermessung 1995"). In order to enable a further use of the LV03-coordinates in combination with new GPS-measurements referencing to LV95 one has to transform the LV03-coordinates into LV95-values. Figure 1 shows the adjustments of such a transformation of the canton or state of Basel-Stadt.

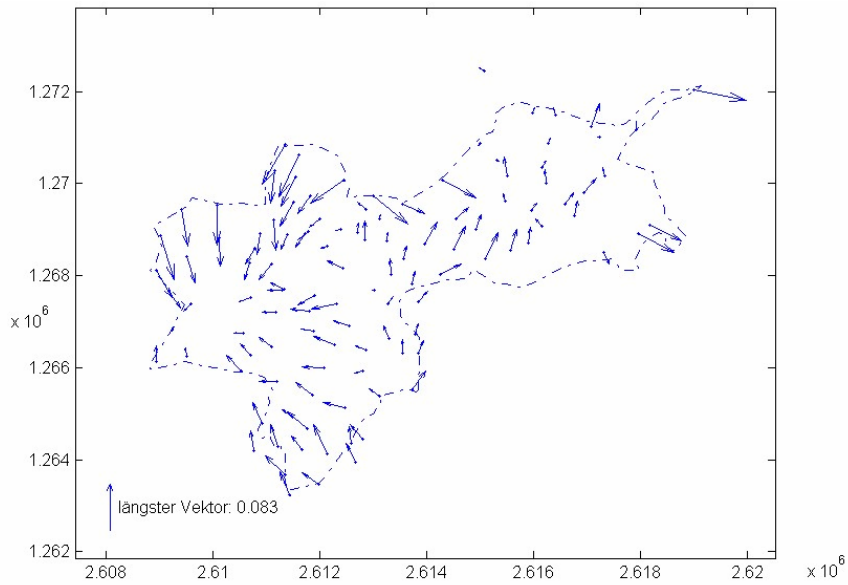

Figure 1. Canton of Basel-Stadt: Adjustments of LV03-coordinates of estimated Helmert transformation LV03 ↔ LV95 based on 114 fiducial points. Length unit [m].

These adjustments show systematic patterns and do not vary randomly. Thus the estimation of a Helmert transformation between the two sets of coordinates should be accompanied by the estimation of the systematic distortions of the old LV03-coordinates. The new GPS-based LV95-coordinates are much more homogeneous than the old ones, which may contain the historical record of their origin, e.g. a systematic error in a baseline measurement, which is carried over to a whole set of following measurements. Furthermore, the Swiss Federal Office of Topography swisstopo ordered the cantons (or states) to divide their territories into triangles in such a way that coordinate transformations fulfill precision requirements in each triangle separately (swisstopo 2000, April 2004, May 2004). How do we determine these triangles? In a triangle the systematic distortion of the LV03-coordinates should not vary too much. Thus one is asking what the systematic distortions of the old coordinates are, and especially of regions with similar distortion. In this paper regions with similar distortion are denoted as "floes" ("Schollen" in German). One has the feeling to identify visually the central zones of such regions, but where should the boundaries be drawn? It is very difficult to assess the variation of the distortion. In addition, from a theoretical point of view the adjustments are known only at the fiducial points.

With the methods of non- and semi-parametric regression the required estimations of Helmert transformations and their systematic distortions can be performed, and interpolation between the fiducial points is practicable. For differentiable estimations, the distortion variation is given by the distortion derivative. Then floes are regions with a nearly vanishing derivative or variation.

2. SEMIPARAMETRIC REGRESSION

2.1 The Model

The standard Gauss-Markov model

$$l = A \cdot x - v \tag{1}$$

with

$$v^T P v = \min \tag{2}$$

assumes normally distributed observation adjustments v. In case we want to estimate the parameters x of a four parameter Helmert transformation $l(t) = A(t)x$, t denotes the start system coordinates, l the target system coordinates, and the matrix A is computed from t. The right side of (1) is composed of the parametric model part $A \cdot x$ and the adjustements v. (1) is a parametric regression description of the data l.

In the case of transforming old 1903 coordinates into GPS-based 1995 coordinates, model (1) is not adequate, because the old coordinates may show systematic distortions relative to the more homogenous GPS-based coordinates. There is no parametric model for such distortions. Therefore we describe the data by the model

$$l(t) = A(t)x + s(t) + n \tag{3}$$

with the so called signal or signal function $s(t)$, and the normally distributed random component n called noise. (2) has to be replaced by

$$\alpha \cdot s^T R s + n^T P n = \min \tag{4}$$

with a positive definite matrix R (Fischer and Hegland 1999 p. 18). A model (3) is also called an additive model (Hastie and Tibshirani 1994 p. 86).

Equation (3) may be interpreted in two ways. In collocation and filtering the model part is given by Ax and the deviation $l - Ax = s + n$ from the model part is split up into the two components s and n, s denoting the systematic part and n denoting the random component. In the semiparametric regression interpretation the functional model is $Ax + s$, consisting of the parametric component Ax and the nonparametric part s. For practical purposes the two interpretations are equivalent. But there is a significant difference from the theoretical point of view. Treating the signal s as a stochastic quantity in collocation and filtering has the difficulty that the systematic deviations described by s have to be interpreted as a realisation of a stochastic process (Moritz 1979 chap. 38), while in semiparametric regression s is a deterministic model component like Ax. The semiparametric regression interpretation is the simpler one.

The regularization parameter α describes the signal to noise ratio, with a positive definite matrix R, called regularizer. The solution of (3) and (4) involves the following problems

- How to describe the nonparametric signal $s(t)$ and how to define the matrix R?
- How to determine α?

The theory is illustrated by a one dimensional regression simulation.

2.2 Description of signal *s* and definition of matrix *R*

The theory see e.g. in (Ruppert, Wand and Carroll chap. 3) or in (Fischer and Hegland 1999 p. 20). The signal

$$s(t) = \frac{1}{\alpha} \cdot \sum_i k_i \cdot b\left(\left|t - t_i\right|\right) \tag{5}$$

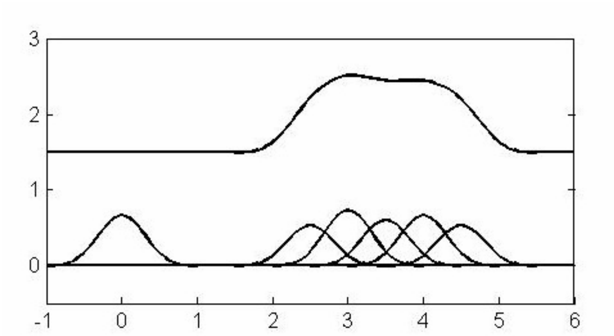

Figure 2. Left below: A B-spline basis function $b(t)$,
right below: shifted and scaled B-Spline basis functions $k_i/\alpha \cdot b(t-t_i)$, i= 1, …, 5,
above: sum $1.5+\sum k_i/\alpha \cdot b(t-t_i)$, 1.5 is added only for the clarity of the figure.

is represented as a linear combination of shifted basis functions b scaled by factors k_i/α, as illustrated by figure 2 for the one dimensional case. The vector k of the coefficients k_i is

$$k = \left(P^{-1} + \tfrac{1}{\alpha} \cdot R^{-1}\right)^{-1} \left(l - A\hat{x}\right), \qquad (6)$$

with $\hat{x}$ denoting the parameter estimates (Fischer and Hegland 1999 p. 19).

In our computations we used the following types of basis functions:

- B-splines of degree 1, 2, 3,
- Gaussian curve.

(5) shows that the differentiability of the basis functions $b(t)$ is carried over to the signal function $s(t)$. This fact will be addressed in 3. when using the derivative $s'(t)$.

Both the regularizer matrix *R* and its inverse R^{-1} are symmetric positive definite, and

$$R_{ij}^{-1} = b\left(\left|t_i - t_j\right|\right) \qquad (7)$$

with a basis function $b(t)$, details see (Fischer and Hegland 1999 p. 20). In calculation, positive definiteness of *R* has to be tested. Too wide basis functions $b(t)$ lead to a violation of that condition.

The transformation at new points t_{new} means computing $Ax + s$ at t_{new}, which involves evaluation of (5) at $t = t_{new}$. Thus prediction in the collocation and filtering sense is reduced to simple interpolation.

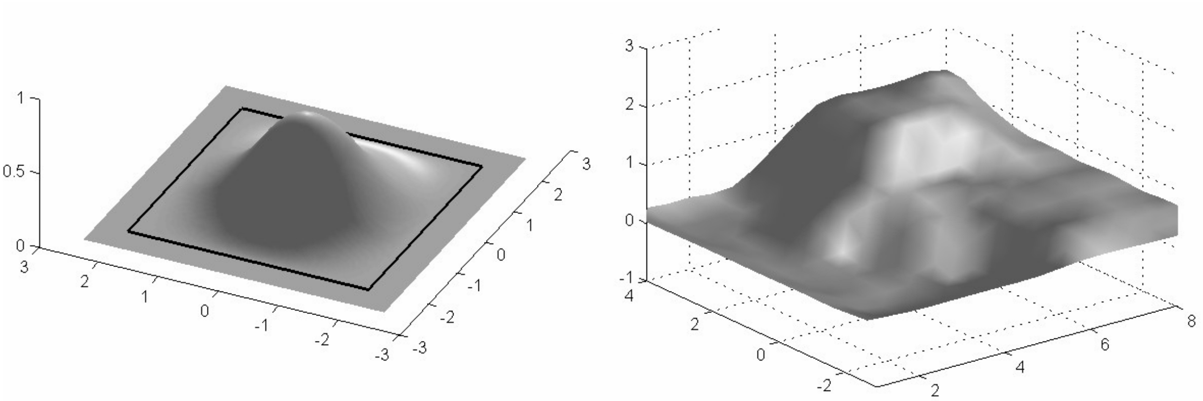

Figure 3. Left: A B-spline basis function of type (7). The square marks the region with values > 0.
Right: A linear combination (8) of shifted and scaled basis functions of type (8).

In case of Helmert transformations t, t_i denote coordinates (x, y), (x_i, y_i) of the start system, and s the signal components (s_ξ, s_η) or non Helmert part of the target system coordinates (ξ,η). Both target system coordinates and their signals are functions of the two start system coordinates. As their basis functions we use tensor products of one dimensional basis functions

$$b(|x-x_i|)\cdot b(|y-y_i|), \tag{8}$$

and both signal components (s_ξ, s_η) at (x,y) are given by

$$s(x,y)=\frac{1}{\alpha}\cdot\sum_i k_i\cdot b(|x-x_i|)\cdot b(|y-y_i|), \tag{9}$$

as illustrated in figure 3.

2.3 Determination of the Regularization Parameter α

The estimations $\hat{x}$, $\hat{s}$ and $\hat{n}$ depend critically on α, as is shown in figure 4 by a simulation of one-dimensional data. A better model estimation is shown in figure 5.

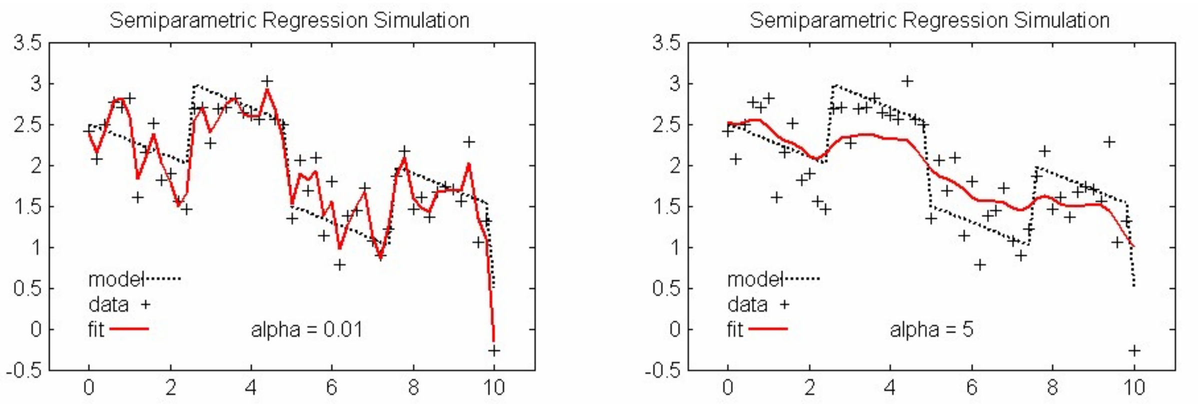

Figure 4. Left: The estimation $\hat{f}+\hat{s}$ for $\alpha=0.01$ is undersmoothed and shows too much variation. Right: The estimation for $\alpha=5$ is oversmoothed.

In the literature, several methods for determining α from the observations are discussed (Ruppert, Wand and Carroll chap. 5, Green and Silverman p. 35, Hansen and O'Leary 1993 p. 1491-95). We only mention the

- L-curve method (with curvature or distance criterion),
- Cross validation and generalized cross validation,
- Morozov's discrepancy method.

For the L-curve method see (Hansen and O'Leary 1993 p.1492, Fischer and Hegland 1999 p. 23). In the Method of Morozov the standard deviation σ of the normally distributed noise n

is given, and α is chosen to fulfil this requirement. This method is used when the mean statistical deviation σ of an observation is known (Hansen and O'Leary 1993 p.1491).

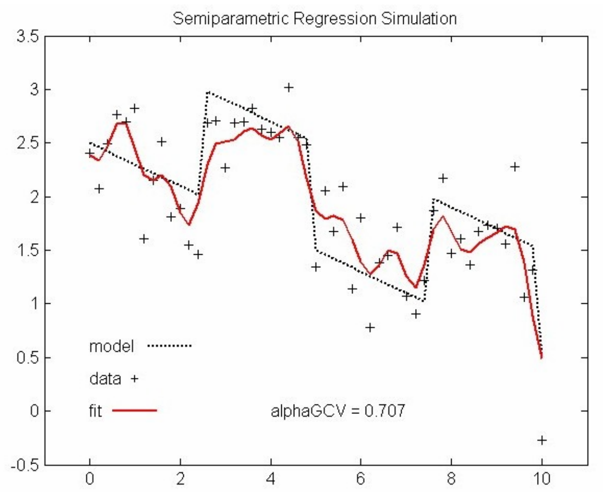

Figure 5. Estimation $\hat{f} + \hat{s}$ for $\alpha_{GCV} = 0.707$.

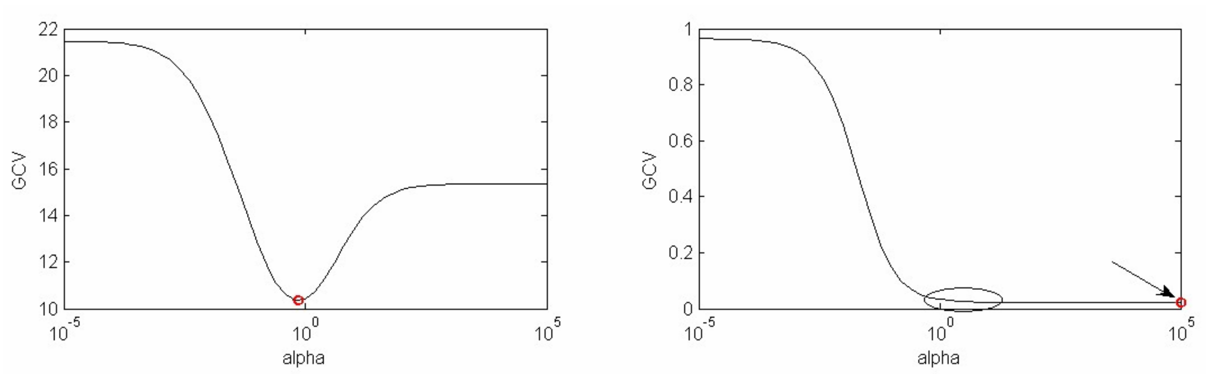

Figure 6: Left: A well defined minimum of GCV(α). Right: GCV(α) with badly defined minimum marked by the arrow. Then α has to be chosen manually in the region marked by an oval, where GCV reaches practically its minimum.

Cross Validation and Generalized Cross Validation are methods justified by a statistical argument. α is chosen in order to minimize the average predictive error of an additional observation (figure 6). Details of the theory see (Green and Silverman 1994 chap. 3.1 – 3.3, Hastie and Tibshirani 1990 chap 3.4). To simplify calculations one usually uses an approximation of CV called Generalized Cross Validation or GCV.

3. DETECTION OF REGIONS OF SIMILAR SIGNAL

3.1 Method

Regions of similar signal $s(t)$ or floes can be identified by inspecting the variation of $s(t)$. We will first treat the one dimensional case. If the signal function $s(t)$ is differentiable, the variation is given by its derivative

$$s'(t) = \frac{1}{\alpha} \cdot \sum_i k_i \cdot \frac{d}{dt} b\left(\left|t - t_i\right|\right), \tag{10}$$

and plotting $s'(t)$ or $\left|s'(t)\right|$ as in figure 7 allows detection of the signal floes of the simulation of figure 5 . A more distinguished picture can be performed when using $\left(s'(t)\right)^2$.

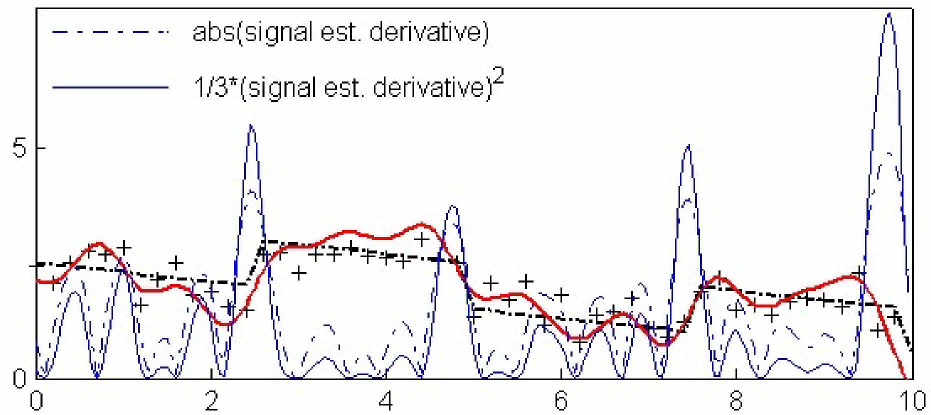

Figure 7. Detection of signal floes using the signal derivative of the GCV estimation. The derivative shows high peaks at the boundaries of the four signal floes, which thus can be detected.

3.2 Helmert Transformation Distortions

The signal estimate of Helmert transformations is composed of the two components $\hat{s}_\xi = \hat{s}_\xi(x, y)$ and $\hat{s}_\eta = \hat{s}_\eta(x, y)$, both being functions of the start system coordinates (x, y). The derivative of the signal estimate is

$$\sqrt{\left|\text{grad } \hat{s}_\xi(x, y)\right|^2 + \left|\text{grad } \hat{s}_\eta(x, y)\right|^2}\,, \tag{11}$$

computed from the squares of the absolute values of the gradients

$$\text{grad } \hat{s}_\xi(x, y) = \begin{bmatrix} \frac{\partial}{\partial x} \hat{s}_\xi(x, y) \\ \frac{\partial}{\partial y} \hat{s}_\xi(x, y) \end{bmatrix}, \quad \text{grad } \hat{s}_\eta(x, y) = \begin{bmatrix} \frac{\partial}{\partial x} \hat{s}_\eta(x, y) \\ \frac{\partial}{\partial y} \hat{s}_\eta(x, y) \end{bmatrix} \tag{12}$$

of the two signal function components. The value of (11) may be plotted as a function of (x, y), e.g. as a surface in space or by using a color representation.

4. APPLICATIONS AND RESULTS

4.1 Triangulation of Coordinate Transformations LV03 ↔ LV95

swisstopo ordered the cantons to define triangulations for coordinate transformation LV03 ↔ LV95 fulfilling precision requirements in each triangle separately (swisstopo 2000, April 2004, May 2004). Signal- and noise estimations are shown in figures 8 and 9.

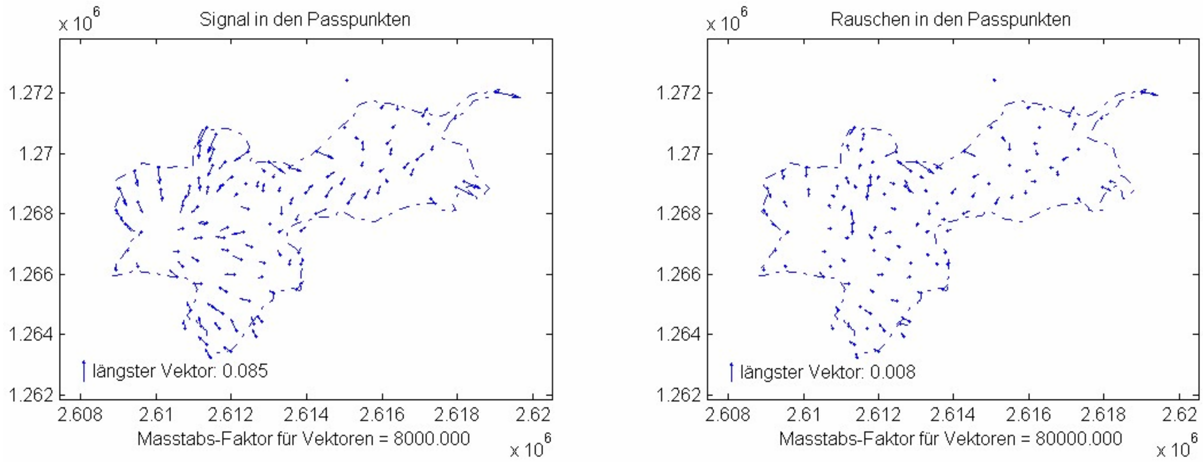

Figure 8. Canton of Basel-Stadt, transformation LV03 → LV95. Left: Signal estimate, right: noise estimate scaled by a factor of 10. Note that the noise varies randomly. Length unit [m].

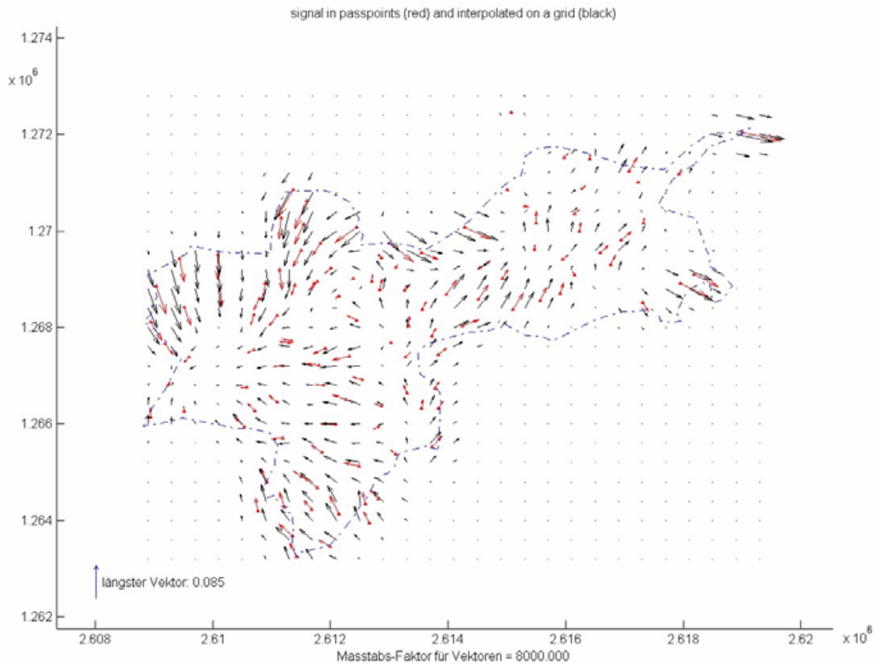

Figure 9. Canton of Basel-Stadt, transformation LV03 → LV95. Signal estimate on fiducial points (red) and its interpolation on a grid (black). Length unit [m].

Figure 9 shows a flow pattern with several floes. Their central parts are marked by regions of signal vectors of similar length and direction. The figure 10 plot of the signal derivative (11) shows the signal variation and allows the identification of the signal floes. In addition to the already known regions I, II and III with large signal variation or strong net deformation, the recently detected region IV (Liechti and Haffner 2006 p.3) could be confirmed.

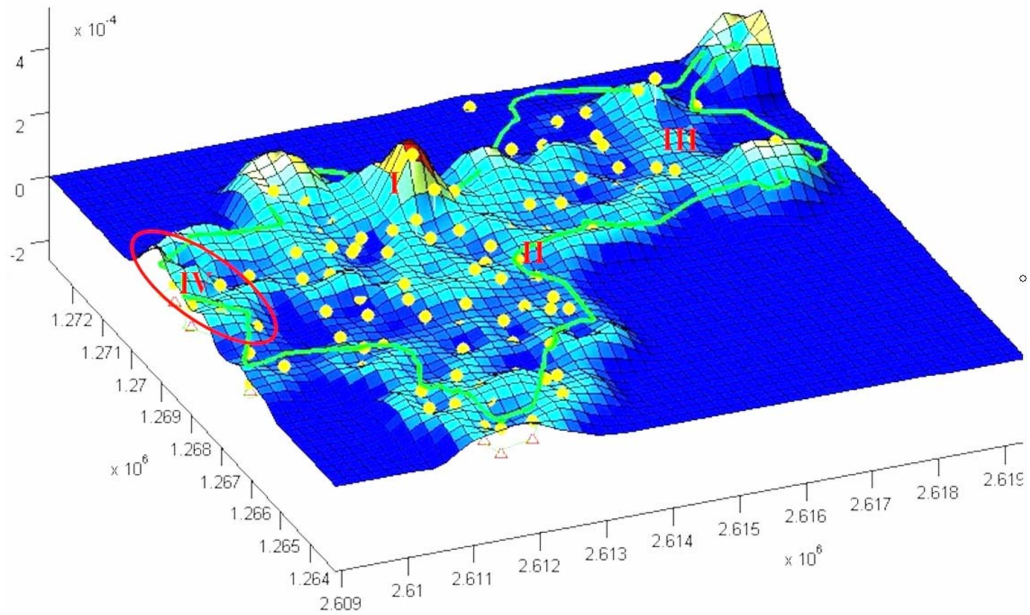

Figure 10: Signal derivative (11) of transformation LV03 → LV95. The points mark the fiducial points, the green line the boundary of the canton of Basel-Stadt. Roman numerals mark regions with large signal variation or strong net tensions.

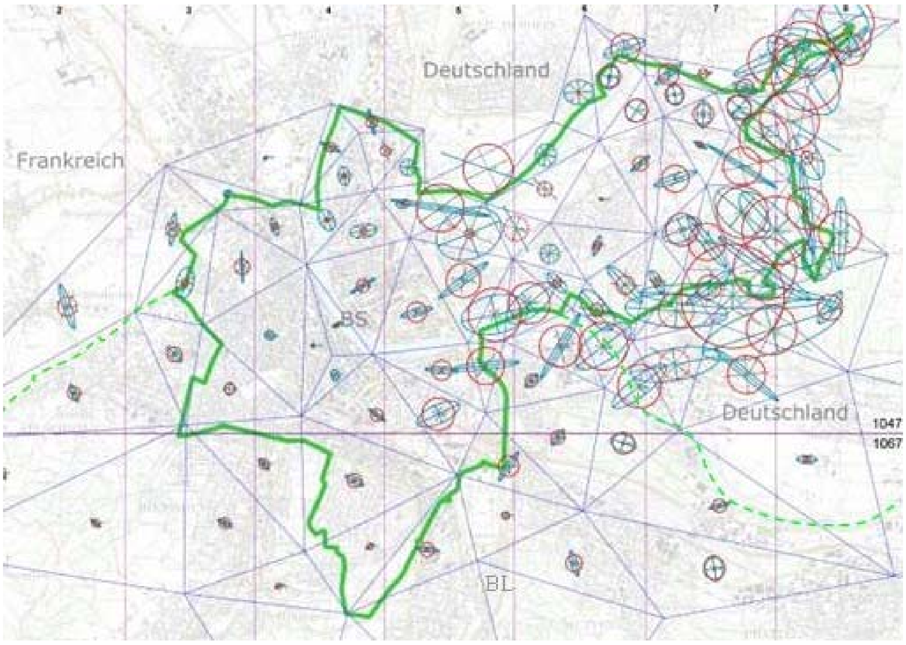

Figure 11. Canton of Basel-Stadt: Triangulation for Transformation LV03 ↔ LV95 with deformation ellipses. With kind permission of Grundbuch- und Vermessungsamt Kt. BS, Switzerland (Liechti and Haffner 2006).

The canton of Basel-Stadt based the layout of its triangles on such estimates of the signal and its derivative of the LV03 ↔ LV95 transformation. Performing signal estimation only on smaller parts of the territory allows refinement of regions with large signal variation. The triangles to be used for transformation LV03 ↔ LV95 are shown in figure 11. Figure 12 shows the projection of a pixel map onto the signal derivative surface. Thus the geographical locations of the net distortions are immediately made visible.

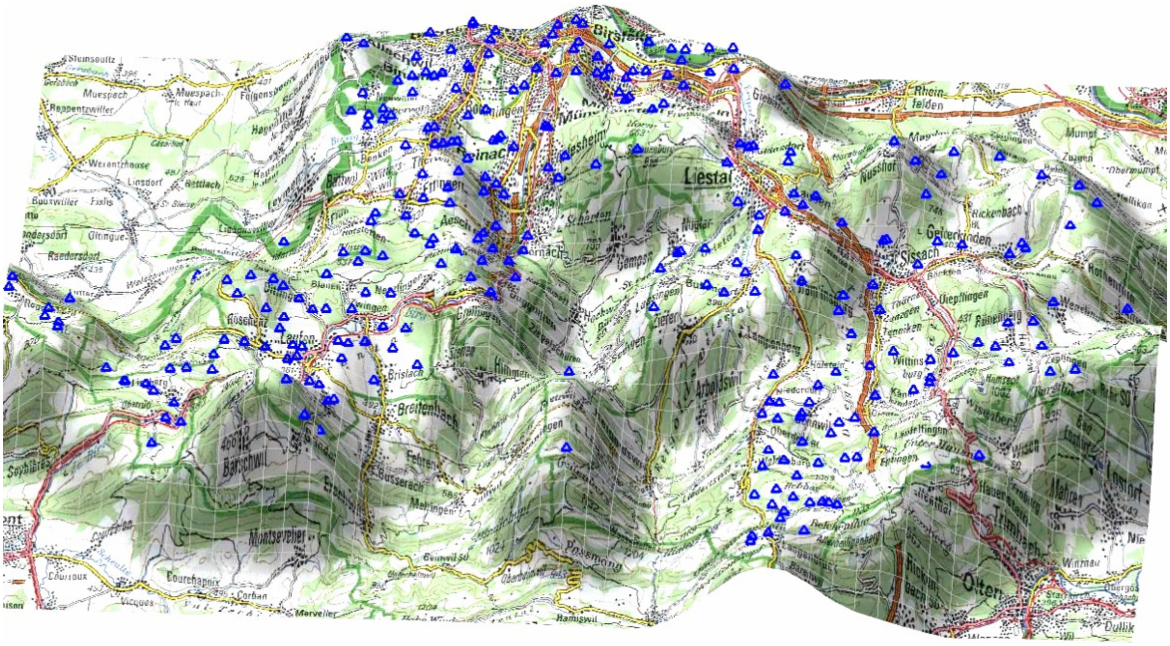

Figure 12. Transformation LV03 → LV95 of canton of Basel-Landschaft: The surface height is the derivative (11) of the signal. This "landscape" has nothing to do with the topography of the region. The fiducial points are marked by blue triangles. Reproduction of the pixel map with kind permission of swisstopo (BA067964).

4.2 Controlling Results

Controlling results involves checking graphically the following criteria concerning the noise:

- Is the choice of α reasonable, more especially: is there a well defined minimum when choosing α by GCV?
- Are each of the estimates $\hat{n}_x$, $\hat{n}_y$ of the two noise components normally distributed ?
- Are the noise azimuths equally distributed?
- Is the quantity $\frac{\hat{n}_x^2+\hat{n}_y^2}{\hat{\sigma}^2}$ ($\hat{\sigma}^2$ is the estimate of σ^2) approximately χ^2 – distributed with two degrees of freedom?
- Do noise signs show a regionally random distribution?
- Are the values of the normalized noise $w_i = \frac{n_i}{\hat{\sigma}}$ reasonable?

The signal behaviour of old coordinates has to be discussed following geodetic aspects.

- Where do the regions with large signal variation lie? E.g. are these regions of flat woodland (e.g. region I of figure 10)?
- What is the history of surveying of the considered region?
- Are there historical hints to distortions of the old net?
- Are there weakly defined fiducial or reconstruction points?
- Did the instruments used produce measurements with systematic deviations?

5. DISCUSSION

The non- or semi-parametric regression is a valuable tool for detection and diagnosis of effects with a non parametric model part. The application to the estimation of systematic net distortions resulted in a useful diagnosis, which gave crucial hints as to how to perform the layout of the LV03 ↔ LV95 transformation triangles in the canton of Basel-Stadt, Switzerland. Other countries deal with similar problems, compare e.g. (Jansa and Augustin 2004). A further application is to model the locally different shifts of a sliding slope over e.g. fifty years:

$$\text{new location} = \text{old location} + \text{shift} + \text{signal} + \text{noise}. \tag{13}$$

The shift describes the mean motion of the slope, the signal the local variation, and the noise the measurement errors. However, when the use of adjusted values is required, methods like multiquadratic interpolation also "interpolating" the noise are to be used.

In the wider context, performing such a signal fit is an ill posed problem with more than one solution. The choice of the regularization parameter α and the regularizer R are arbitrary in the sense that one may choose the method of determining α and the basis function $b(t)$, the latter restricted by differentiability requirements and the demand that R be positive definite. These choices imply that it has to be emphasized that this method is rather reproductive and automatic than objective (Green and Silverman 1994 p.29).

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REFERENCES

Bruehlmann, P.: Koordinatentransformationen: Nichtparametrische Regression und Schollendetektierung. Diploma thesis, dept. of geodesy, FHBB, 2004 (not published)

Fischer, B. and Hegland, M.: Collocation, Filtering and Nonparametric Regression. ZfV 124(1), S. 17-24, 124(2), S. 46-52.

Fischer, B., Schwaninger, M. and Sievers, B.: Nichtparametrische Regression am Beispiel der Helmerttransformation. Geomatik Schweiz 6/2003, S. 346-47

Green, J.P. and Silverman, B.W.: Nonparametric regression and generalized linear models. Chapman and Hall, London 1994.

Hansen, P. and O'Leary, D.P.: The use of the L-curve in the regularization of discrete ill-posed problems. SIAM (Society of Industrial and Applied Mathematics) J. Sci. Comp. 14(6): 1487-1503, 1993.

Hastie, T.J. and Tibshirani, R.J.: Generalized additive models. Chapman and Hall, London 1990.

Jansa, J. and Augustin, L.: Ein Vorschlag fuer die Homogenisierung des oesterreichischen Fixpunktnetzes. Vermessung & Geoinformation 2/ 2004, p. 59 – 67.

Liechti, H.U. and Haffner, P.: Dreiecksvermaschung Kanton Basel-Stadt, Technischer Bericht, Basel 2006 (not published).
Moritz, H.: Advanced physical geodesy, 2nd ed.. Wichmann, Karlsruhe 1989.
Ruppert, D., Wand, M.P. and Carroll, R.J.: Semiparametric Regression, Cambridge University Press 2003.
Schwaninger, M.: Nichtparametrische Regression: Softwareentwicklung. Diploma thesis, dept. of geodesy, FHBB, 2002 (not published)
swisstopo: Transformation LV03 ↔ LV95, Bereitstellung der Transformationsgrundlagen für die AV, Weisungen betreffend die Erstellung der kantonalen Dreiecksvermaschung, 2000.
swisstopo: Transformation LV03 ↔ LV95, Bereitstellung der Transformationsgrundlagen für die AV, Empfehlungen bezüglich der Qualitätsanforderungen bei der Erstellung der Dreiecksvermaschung durch die Kantone, April 2004.
swisstopo: Transformation LV03 ↔ LV95, Bereitstellung der Transformationsgrundlagen für die AV, Technische Hinweise zur Erstellung der Dreiecksvermaschung, Mai 2004.

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