Centre des Techniques Spatiales

Division de Géodésie Spatiale

Analysis of DORIS Stations Coordinates Time Series by the Singular Spectrum Analysis (SSA)

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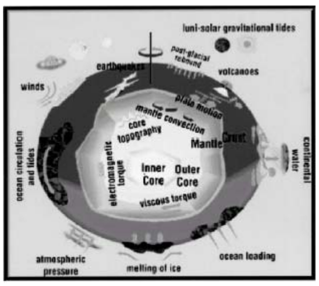
Introduction

EARTH : complex system; siege of temporal variations

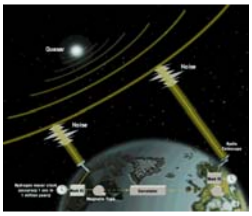
SPACE GEODESY : science which uses measurements of the artificial satellites which turn around the earth to determine the shape of the earth and its changes in time :

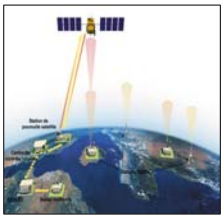
- ➔ Gravity field
- ➔ Geoid (mean sea level)
- ➔ Orientation of the Earth
- ➔ Deformation of the earth's crust
- ➔ …



Introduction

Techniques of space geodesy : VLBI, SLR, GPS, DORIS





VLBI

Measures evolution of the phase of the incidental wave (even radiosource) between two radio telescopes.

SLR

Measures the time intervals required for pulses emitted by a Laser transmitter (station) to a satellite.

GPS

Measures satellite- receiver distance deduced from the time between emission and reception.

DORIS

Measures Doppler effect.

Introduction

The richness and the great quantity of the measurements collected by these systems (VLBI, SLR, GPS and DORIS) Allows today to represent the displacement of the stations on the Earth in the form of time series of coordinates which require the development of the adequate methods of analysis for a better exploitation.

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Introduction

Statistical methods for the study of the time series in space geodesy

➢ **Signal :**

– Fourier Spectrum
– Wavelets (Daubechies 1988, Mallat 1989, Johnstone & Silverman 1997)

➢ **Noise :**

– Allan variance (Time and frequency, Allan 1966)
– Spectral density and maximum likelihood estimate (Agnew 1992, Langbein & Johnson 1997, Zhang et al. 1997, Mao et al.1999, Blewitt & Lavallée 2002, Williams 2004)

➢ **Method suggested :**

– **Singular Spectrum Analysis** (Climatic time series, Ghil et al. 2002)

Objectives

Application of the SSA (Singular Spectrum Analysis) method in the analysis of time series of stations coordinates of space geodesy, in order, to collect the maximum of exploitable information on the signals and their noises which will make it possible to better apprehend the temporal variability of stations movement.

Time series

Time Series :

Sequence of numerical observations X_t measured at successive times **t** (t = 1…, *N*; *N* : length of the series)

$$X_t = m_t + s_t + \varepsilon_t$$

m_t : ***Trend*** *(long-term evolution of the series)*

s_t : ***Cyclical component*** *(seasonal effect of the series)*

ε_t : **Residual component** *(noise affecting the series)*

Singular Spectrum Analysis

Objective : **Extraction** of the **significant components** from a time series (trend, seasonal component and noise).

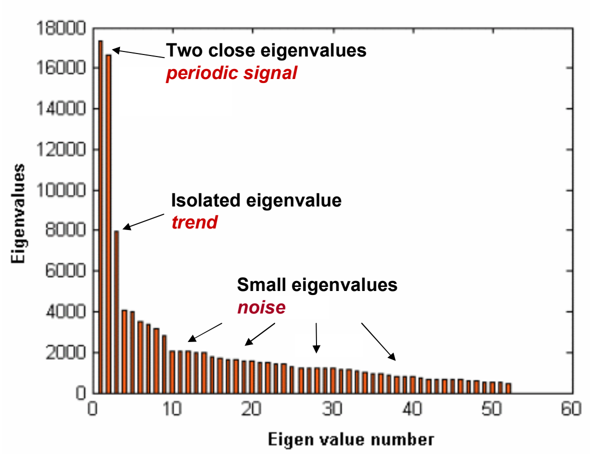
Principle : **Computation** of the **eigenvalues** and **the eigenvectors** of the **lag-covariance matrix C** formed from the time series $\{X_t$, t=1...,N$\}$ and the **reconstruction** of this time series **on the basis of the principal eigenvectors**.

$$C = \frac{1}{N-M+1} D^t D \qquad D = \begin{pmatrix} X_1 & X_2 & \cdots & X_M \\ X_2 & X_3 & \cdots & X_{M+1} \\ \vdots & \vdots & \ddots & \vdots \\ X_{N-M+1} & X_{N-M+2} & \cdots & X_N \end{pmatrix}$$

M : Lag of covariance ; N : length of the series.

Singular Spectrum Analysis

Eigenvalues ⇨ *partial variance in the direction of the associated eigenvectors*



Data Used

➔ Weekly series of the residual coordinates of (01) station DORIS (SYPB) provided by the IGN/JPL analysis centre.

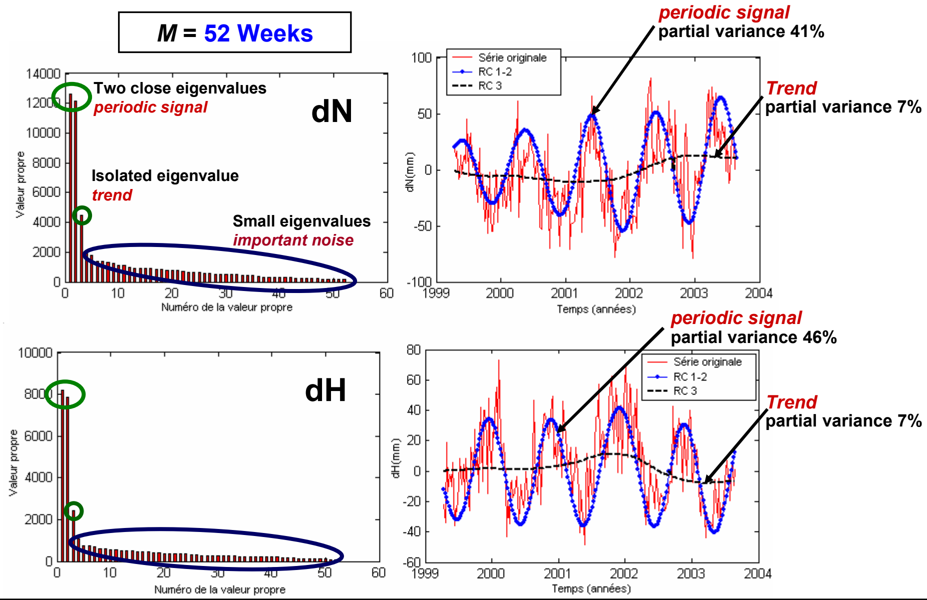
Station acronym	Site	Country	Data span (years))	Period (years))	Observations number
SYPB	Syowa	Antarctique	1999.2 - 2003.6	4.4	225

➔ Expressed in the **Local Geodetic Reference Frame** :

(**dN** : North, **dE** : East , **dH** : Up)

Results and analysis

M **= 52 Weeks**



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Results and analysis

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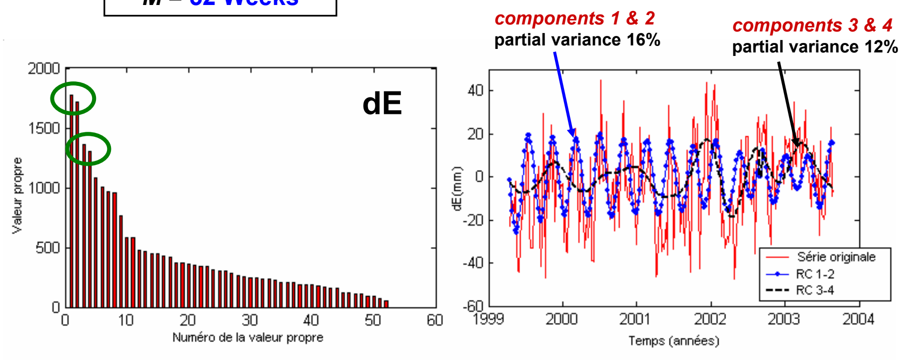

- **Two pairs of close eigenvalues ⇨ two periodic components**
- **The reconstructed components 1-2 and 3-4 ⇨ non periodic series**

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Results and analysis

Series	Reconstructed Components	Partial variance	Standard deviation
dN	RC 1-2	41	33
	RC 1-2 & RC 3	48	35
	RC 1-2, RC 3 & RC 4-5	54	37
dE	RC 1-2	16	12
	RC 1-2 & RC 3-4	27	15
	RC 1-2, RC 3-4 & RC 5-6	37	17
	RC 1-2, RC 3-4, RC 5-6 & RC 7-8	45	20
	RC 1-2, RC 3-4, RC 5-6, RC 7-8 & RC 9	49	21
	RC 1-2, RC 3-4, RC 5-6, RC 7-8, RC 9 & RC 10-11	54	22
dH	RC 1-2	46	25
	RC 1-2 & RC 3	53	27
	RC 1-2, RC 3 & RC 4	56	27
	RC 1-2, RC 3, RC 4 & RC 5,6	60	28

- **Denoising ⇨ extract from the initial series, the series reconstructed on the basis of principal components (true signal).**
- **For denoising ⇨ ? number of the principal components to take into account.**

Results and analysis

Series		Minimum	Maximum	Mean	STD
(dN)	Original series	-79	82	-4	34
	Denoised series	-54	79	1	37
(dE)	Original series	-47	45	-4	20
	Denoised series	-53	57	0	22
(dH)	Original series	-48	73	4	26
	Denoised series	-49	56	-1	28

The standard deviation (STD) of the initial series was augmented after the reconstruction which remains dependent on the number of the reconstructed components representing the true signal.

Conclusion

The application of method SSA to the weekly series of sets of residual coordinates of the station DORIS (SYPB), permits to better highlight the trend and the periodic signal. Indeed, we have clearly identify and quantify a periodic signal at one year and a light trend for the two series (dN) and (dH).

To extract the noise from the original signal, this method presents difficulties of the determination of the number of eigenvalues to take into account to represent the true signal. The results obtained showed that these series are affected by a significant noise and that the increase of principal reconstructed components (true signal) involved the augmentation in the standard deviation of the series denoised compared to original series.

Thank you