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Verification of a two-stage valuation model for different types of real estates in local markets

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INTRODUCTION

The process of determining market or cadastral value of a real estate includes nowadays the application of more or less complicated algorithms usually basing on a relatively small number of market data. Even when the number of market data is important and allows to estimate the parameters of multidimensional functional models, achieved in such a way model value of the valuated real estate constitutes at the same time the prediction of its value. Considering a large diversification of input data, such prediction is charged with a considerable standard deviation, which reduces its reliability.

Another method used in practice, to determine the market value is based on a more coherent market database, which implies that the number of data is very considerably restrained. In such a situation, there is no possibility to estimate the accuracy of the final prediction. Even when we succeed to get statistically more accurate estimation of the predicted value, we still have doubts if the result is representative in relation to the whole analysed market of real estates.

Consequently, the proposed two-stage model for determining a real estate value seems to be an optimal solution. It satisfies the condition that input data used at the first stage are representative in consideration of their review character and large number and, at the same time, allows at the second stage, to precise a real estate model value by correcting it on the basis of the parameters obtained from a coherent, strongly confined database having features very close to the valuation object.

TESTED VALUATION MODELS

In the process of modelling the real estate market values, additive (1) or multiplicative (2) functions can be used in form:

$$w = a_0 + \sum_{k=1}^{m} g_k\left(x_k\right) \tag{1}$$

$$w = a_0 \cdot a_1^{x_1} \cdot a_2^{x_2} \cdot \cdots \cdot a_m^{x_m} \tag{2}$$

where:

w – unit price or value of real estate,
x_k – value of attribute *k* for real estate,
g – function of real estate price - attribute *k* relation,
a_0 – free term in the model (unit value of a real estate, for zero of all attributes),
a_j – estimated model parameters.

In the general structure of an additive model (1), we will find polynomials of different degrees as functions *g*. Thus, developing the model (1), we have:

$$w = a_0 + \sum_{k=1}^{m} (a_{k_1} \cdot X_k + a_{k_2} \cdot X_k^2 + ... + a_{k_{n_k}} \cdot X_k^{n_k}) \qquad (3)$$

where:

m – number of attributes considered in the model,
n_k – degree of polynomial for *k*-th attribute.

At polynomial forms of the function *g*, the whole model maintains the linearity in relation to the parameters. Therefore, the estimation of the model parameters may be done the same way as in the case of multiple regression.

Estimation of parameters of both valuation model forms can be done by the least squares method. A detailed estimation algorithm provided with the accuracy analysis and the description of statistical methods to verify estimated model is presented in ref.

<u>ISOLATING THE SYSTEMATIC FACTOR AND THE RANDOM COMPONENT OF THE MODEL</u>

Estimated valuation model leads to isolating its systematic factor from unit prices of real estates. They are predicted values of real estate prices *W* determined using the model. Differences between real market prices assembled in database *C* and model values *W* are model residuals:

$$[\delta] = [C] - [W] \qquad (4)$$

where:

$[\delta]$ - valuation model residuals,
$[C]$ - vector of real estate prices in data base,
$[W]$ - vector of real estate model prices.

To each systematic factor corresponds a set of model random residuals δ having the inaccuracy characteristics contained in its covariance matrix $Cov[\delta]$:

$$[\delta]=[C]-[X]\cdot[X]^{+}\cdot[C]=\left[I-X\cdot X^{+}\right]\cdot[C] \qquad (5)$$

$$Cov[\delta]=\sigma_0^2\cdot\left[I-X\cdot\left(X^T\cdot X\right)^{-1}\cdot X^T\right] \qquad (6)$$

where:

X^{+} – pseudo-inverse of matrix X,

σ_0^2 – model residual variance.

From the residuals for selected real estates and from their covariance matrix, a random correction for the prediction of estimated real estate market value is calculated.

PREDICTION OF REAL ESTATE MARKET VALUE

Point estimation of real estate model value

Basing on the parameters of a selected model and on the attributes of an estimated real estate, we determine its model value w_M with full accuracy analysis. The analysed real estate value determined in such a way, being a prediction of estimated model, can be its probable market value. In the case of an additive model in multiple regression form, the prediction of value of real estate selected from a given market, is performed according to the following formula:

$$w_M=\begin{bmatrix}1 & x_1 & x_2 & \dots & x_m\end{bmatrix}\cdot\hat{a} \qquad (7)$$

where:

$\begin{bmatrix}1 & x_1 & x_2 & \dots & x_m\end{bmatrix}$ – vector of values of estimated real estate attributes,

$\hat{a}$ – vector of estimated model parameters.

The accuracy of such a prediction is evaluated by its variance expressed by formula:

$$\sigma^2(w_M) = [1 \quad x_1 \quad x_2 \quad \ldots \quad x_m] \cdot Cov(\hat{a}) \cdot [1 \quad x_1 \quad x_2 \quad \ldots \quad x_m]^T \tag{8}$$

where:

$Cov(\hat{a})$– covariance matrix of vector model parameters.

Selection of real estates most similar

Applying a selected qualitative method, from the database used to estimate model parameters, we choose the group of *k* real estates most similar to the estimated one.

Correction (making real) of real estate model value

From the vector random component we isolate residuals δ_{i_w}, corresponding to selected most similar real estates, and from the residuals covariance matrix – a sub-matrix $Cov[\delta_w]$ having $(k \times k)$ dimensions and containing elements that correspond to the isolated residuals.

From the systematic model we have determined the real estate model market value w_M with its standard deviation $\sigma(w_M)$. Then, from the random model we estimate the market value of the real estate random component:

$$w_L = \left[\underline{1}^T P \underline{1}\right]^{-1} \cdot \underline{1}^T P [\delta_w] \tag{9}$$

where:

$\underline{1} = [1 \quad 1 \quad \ldots \quad 1]$ – vector containing ones of dimensions

with the weight matrix being covariance matrix inverse:

$$P = Cov^{-1}[\delta_w] \tag{10}$$

The standard deviation of the random component $\sigma(w_L)$ is calculated according to the formula:

$$\sigma^2(w_L) = \sigma^2_{0_w} \cdot \left[\underline{1} \cdot P \cdot \underline{1}^T\right]^{-1} \qquad (11)$$

where residual variance $\sigma^2_{0_w}$ is calculated for the group of k selected real estates by formula:

$$\hat{\sigma}^2_{0_w} = \frac{\delta_w^T \cdot P \cdot \delta_w - w_L \cdot \underline{1}^T \cdot P \cdot \delta_w}{k-1} \qquad (12)$$

The final, corrected market value of the estimated real estate is calculated as follows:

$$w_{M+L} = w_M + w_L \qquad (13)$$

The addition of correction w_L to model value w_M makes the prediction of real estate value more precise by taking into consideration the data of the real estates most similar to the estimated one.

Between variances of prediction of a real estate market value, obtained from the model w_M as well as the prediction corrected with the random factor w_{M+L}, the following relation occurs:

$$\sigma^2(w_{M+L}) = \sigma^2(w_M) - \sigma^2(w_L) \qquad (14)$$

PRACTICAL IMPLEMENTATION OF TWO-STAGE MODEL

Presented above proceeding algorithm was verified on many different local markets for different types of real estates. Every time it brought about improving the market value of the estimated real estate predicted by model, specifying its value by adding a correction.

In the tables below, are presented some of model estimation results in additive (linear in consideration of independent variables and parameters or in relation to the parameters only) and multiplicative form. We present the results of valuating a dwelling in southeast Poland. Representative databases of 68÷100 real estates, gathered on different local markets, constituted preliminary data. Acquired information on transactions concern two towns in Poland, diversified in respect of factors shaping the prices of real estates. Different valuation models – additive or multiplicative – have been used in particular cases.

Table 1. Results of additive and multiplicative models estimation

	additive linear model		multiplicative model	
	Wieliczka town	Rzeszów town	Wieliczka town	Rzeszów town
n	92	98	93	100
u	11	13	10	13
σ_0	207,78	227,12	0,077	0,101
V	0,08	0,09	0,07	0,09
R^2	0,89	0,75	0,91	0,73
s	0,60	0,75	0,56	0,67
a_0	**752,176**	**2026,47**	**7,053**	**7,524**
a_1	**0,58**	**-0,34**	**0,61**	**-0,27**
a_2	**0,30**	**0,33**	-0,05	**0,35**
a_3	-0,08	-0,12	**-0,11**	-0,10
a_4	-0,29	-0,10	0,00	-0,12
a_5	**0,66**	**0,12**	0,00	**0,13**
a_6	**-0,30**	**-0,49**	**0,22**	**-0,51**
a_7	0,08	0,10	0,00	0,13
a_8	**0,21**	**0,59**	**0,12**	**0,59**
a_9	**0,24**	**0,56**	**0,06**	**0,53**
a_{10}	0,07	**0,36**		**0,34**
a_{11}		**-0,27**		**-0,26**
a_{12}		**-0,17**		-0,12

Final valuation results for a dwelling inWieliczka town:

model in additive form:

Model value of a real estate:	799,90 [EUR/m2]
Standard deviation of the model value:	25,02 [EUR/m2]
Random value:	- 26,54 [EUR/m2]
Standard deviation of the random value:	19,36 [EUR/m2]
Final prediction of the market value:	773,36 [EUR/m2]
Standard deviation of the final prediction:	15,85 [EUR/m2]

model in multiplicative form:

Model value of a real estate:	761,64 [EUR/m2]
Standard deviation of the model value:	21,49 [EUR/m2]
Random value:	- 20,93 [EUR/m2]
Standard deviation of the random value:	11,62 [EUR/m2]
Final prediction of the market value:	740,70 [EUR/m2]
Standard deviation of the final prediction:	18,08 [EUR/m2]

Final valuation results for a dwelling in Rzeszów town:

model in additive form:

Model value of a real estate:	523,03 [EUR/m2]
Standard deviation of the model value:	31,20 [EUR/m2]
Random value:	- 31,59 [EUR/m2]
Standard deviation of the random value:	16,93 [EUR/m2]
Final prediction of the market value:	491,44 [EUR/m2]
Standard deviation of the final prediction:	26,21 [EUR/m2]

model in multiplicative form:

Model value of a real estate:	507,38 [EUR/m2]
Standard deviation of the model value:	31,54 [EUR/m2]
Random value:	- 30,88 [EUR/m2]
Standard deviation of the random value:	14,77 [EUR/m2]
Final prediction of the market value:	476,50 [EUR/m2]
Standard deviation of the final prediction:	27,86 [EUR/m2]

CONCLUSIONS

On the basis of presented examples, we can see that in the case of all valuations, the standard deviation of the final prediction of the real estate value constitutes 2-10% of its altitude, being always minimum over a dozen percent less then the standard deviation of real estate predicted value got directly from the model. Then, the results achieved are considerably more precise than those obtained using only a function model.

Presented two-stage valuation model aims to make more precise the real estate value determined using a valuation model well fitted to the local market and statistically verified. The advantage of the algorithm is the selection in a database of similar real estates, used to estimate parameters of a multidimensional function model, a group of real estates most similar to the valuated object and to employ them in "correcting" a model value of a real estate.

Thank you very much for your attention