On the augmentation of Israeli GPS-BM data with a global Earth geoid model

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Integrating the Generations
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Presentation outline

- **Introduction to the official geoid undulation model in Israel**
- **Comparison of the preliminary Earth Global Geoid Model (EGM07) to a GPS-BM dataset**
- **Co-kriging, a geostatistical method for data integration**
- **Experimental results**
- **Summary and conclusions**

The official geoid undulation model in Israel

- The Survey of Israel (SOI) came to the conclusion that there is no justification to maintain a countrywide vertical orthometric control network.
- A combination of ellipsoidal heights with an official geoid undulation model (OGUM) will serve as a substitute to the countrywide vertical orthometric control network.
- An important aspect of a proper OGUM is its ability to represent undulation change over a distance.

The official geoid undulation model in Israel

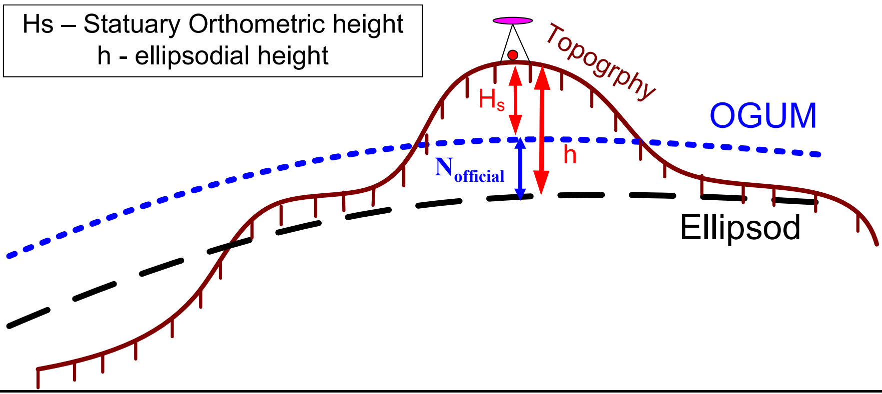
- As of May 2007 surveyors in Israel enjoy the ability to define statutory orthometric heights in real time using a single GNSS receiver equipped with the Israeli official geoid undulation model (ILUM).
- Instead of occupying at least 4 benchmarks, they can use just one benchmark for checking purposes only.
- **ILUM is actually the countrywide orthometric height reference system of Israel.**

Using the official Geoid undulation model

$N_{official}$ is the official geoid undulation as published by the national geodetic and mapping authority

$Hs = h - N_{official}$

Hs – Statuary Orthometric height
h - ellipsodial height

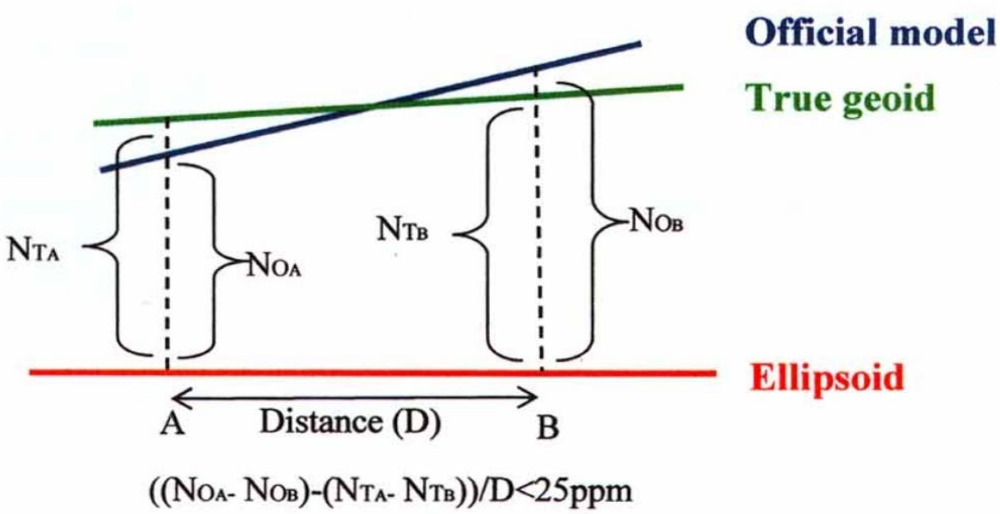


The Need for Accurate Geoid

- The professional literature stress the need for a countrywide reference system for (orthometric) height measurements.
- These orthometric heights using GNSS and geoid-model should agree with the registered heights of existing bench-marks.
- It is stated that there is a need for a 1 cm geoid model.
  - **Is it really achievable?**
  - **Is it really necessary?**

- Various specifications stress the need for high absolute accuracy. This need is **overemphasized** since relative accuracy is more important.
- It was suggested that the statewide official geoid may have lower accuracy. This will require surveyors to establish islands of high accuracy orthometric heights for special projects.

Difference between OGUM and the True geoid



Evaluation of geoid undulation models in Israel

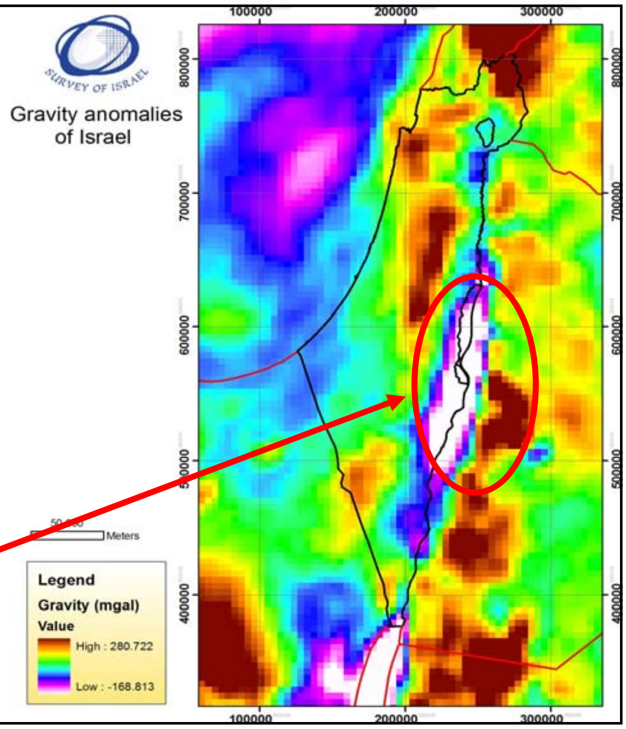
- The official Israeli undulation model (ILUM) is based on 834 points with given ellipsoidal and orthometric heights. Kriging, a Geostatistical approximation method, was used for the construction of a geoid undulation surface. The geoid undulation values were calculated on a grid with a resolution of 0.5 x 0.5 km.

- The preliminary Earth Gravity Model (EGM07)was developed by the US National Geospatial Agency by optimally combining gravitational information extracted from dedicated geopotential mapping satellite missions (CHAMP, GRACE), with data from a global gravity anomaly database at a 5′x5′ resolution. This new model is completed to degree and order of 2160.

The Global Earth Gravity Model in Israel

- Resolution 2’×2’
- Created from Grace based satellite data

Large anomalies in the gravity field near the great rift area.



The Global Earth Geoid Model in Israel

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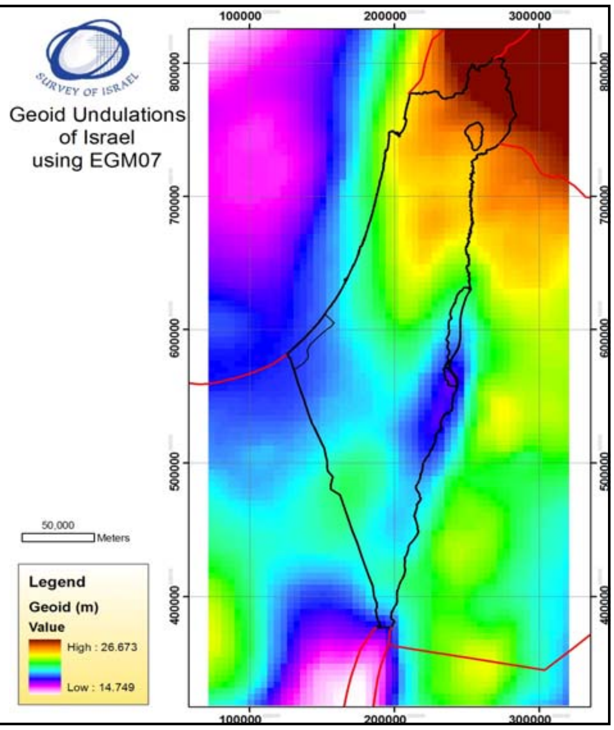
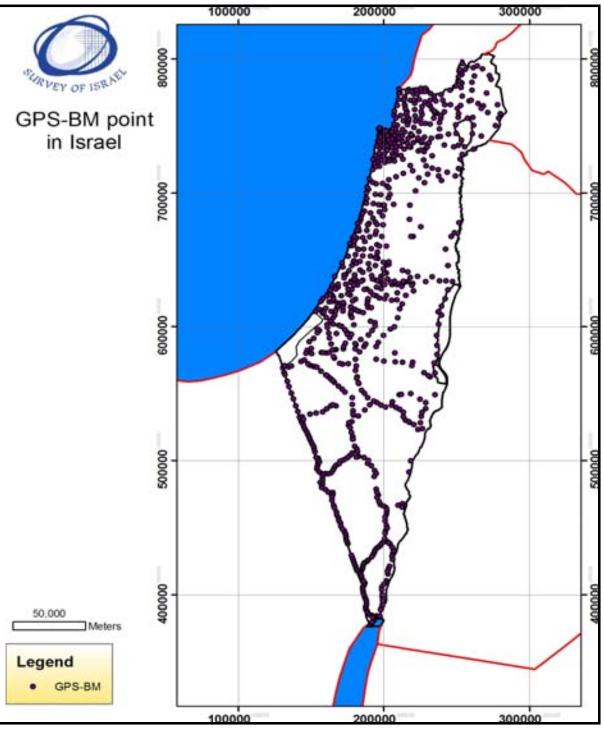


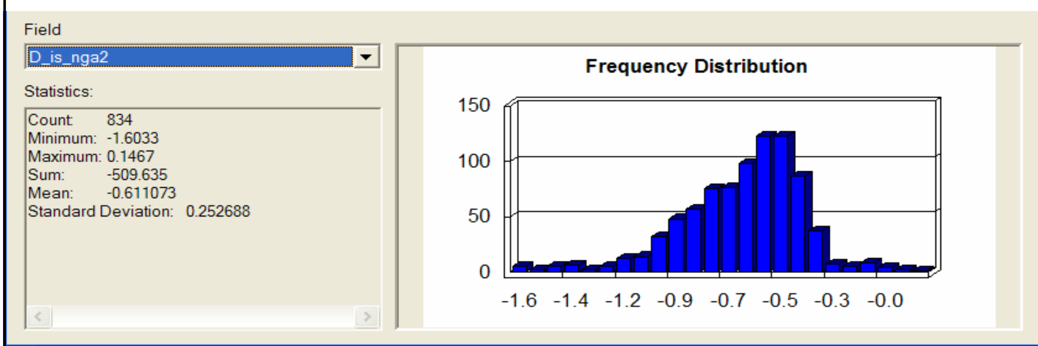
GPS-BM measurements in Israel

- 834 GPS/leveling measurements made with high accuracy.
- The accuracy of the geoid model is low inside the leveling loops where there is a void



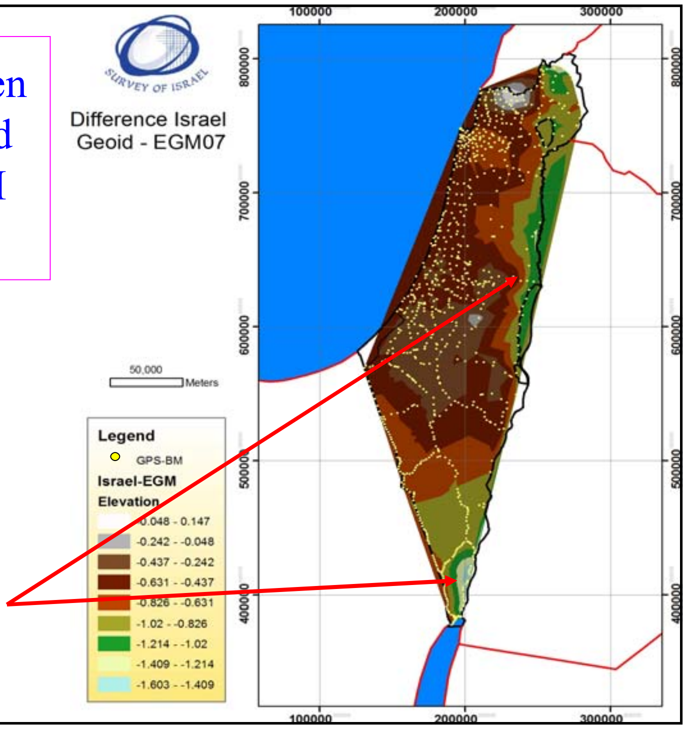
Comparing the EGM07 geoid with local GPS-BM points

- 834 points were tested.
- Datum shift of -0.61 and a standard deviation of 0.25 m.



Difference between the EGM07 geoid and the GPS-BM points

- Large difference observed along the Great Rift (African-Syrian fault) were the low resolution data was unable to follow the large anomalies

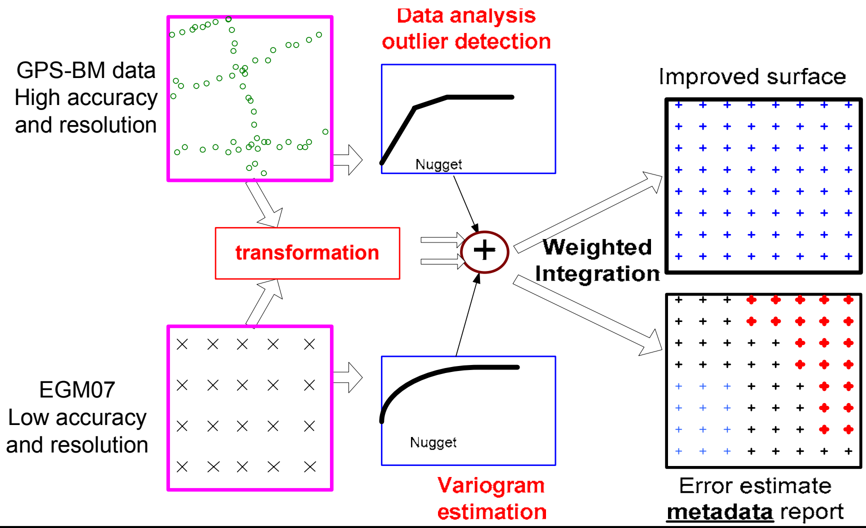


Experiment of enhancing the local geoid model using a global geoid
An inexpensive approach

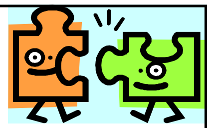
- The goal of the following discussion is to improve the local geoid using the global geoid.
- This will create an enhanced dataset with good accuracy in the leveling-loops voids.
- Geostatistical methods are used for this data integration as applied in ArcGIS and using some developed utilites.

Augmentation of the global dataset with GPS-BM data

The GPS-BM data has large gaps while the EGM has low accuracy



Ordinary Cokriging



Standard Co-kriging use information contained in secondary variables in the prediction of a primary variable.

Given a set of n observed data points $z(s_1), z(s_2),\ldots, z(s_n)$, at locations $s_1, s_2,\ldots, s_n$, and let $y(s_1'),\ y(s_2'),\ \ldots,\ y(s_m')$ the secondary variables measured at $\boldsymbol{m}$ observations at locations $s_1', s_2',\ldots, s_m'$.

(1)

$$\tilde{z}(s_0) = \lambda_1 \cdot z(s_1) + \ldots + \lambda_n \cdot z(s_n) + \kappa_1 \cdot y(s_1') + \ldots + \kappa_n \cdot y(s_m')$$

Data set 1 **Data set 2**

Where $\lambda_1,\ldots, \lambda_n$, and $k_1,\ldots,k_n$ are the weights which are calculated such that $Z(s_0)$ is the best linear unbiased prediction

How to compute the Cokriging coefficients?

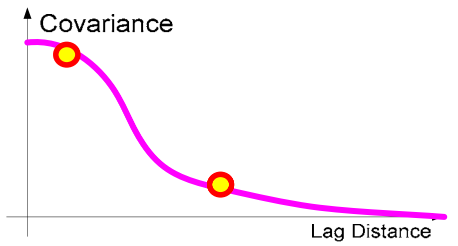
- Coefficients are computed by solving the following equation:

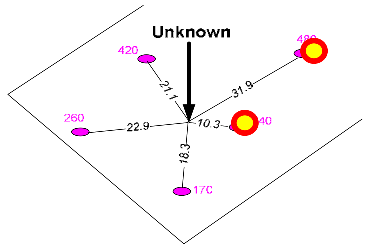
$$\begin{bmatrix} C(z,z) & C(y,z) & I & 0 \\ C(z,y) & C(y,y) & 0 & I \\ I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \bar{\lambda} \\ \bar{\kappa} \\ \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} c[z, z(s_0)] \\ c[z, y(s_0)] \\ 1 \\ 0 \end{bmatrix}$$

Where C(z,z) and C(y,y) are the covariance matrices, C(z,y) and C(y,z) are the cross-covariance matrices computed using the TLS.

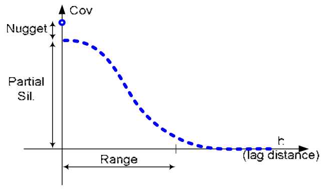
First step in co-kriging
Variogram / Covariance function estimation

- Spatial dependence is usually expressed mathematically in the form of a spatial coherency function such as the semi-variogram, or the covariance function.
- The semi-variogram and the covariance function are valuable tools in explanatory data analysis. Moreover, these functions control the way in which kriging weights are assigned to data points during interpolation.





Computing the Covariance function



- The **covariance** is a measure of similarity between two signals

$$c_{zy}(h) = E(z(s_i + h) \cdot y(s_i)) - m_z \cdot m_y$$

- The Spherical model was found to be most suitable for the specific geoid undulation data. This model is popular in geo-statistics but it is different than the traditionally used exponential or Gaussian models:

$$\begin{cases} \sigma_e^{\,2} + \sigma_y^{\,2}\left[1 - 1.5\left(\frac{h}{r}\right) + 0.5\left(\frac{h}{r}\right)^3\right] & 0 \le h \le r \\ \sigma_e^{\,2} & h > r \end{cases}$$

- The range, sill and nugget parameters were determined using WLS together with graphical methods.

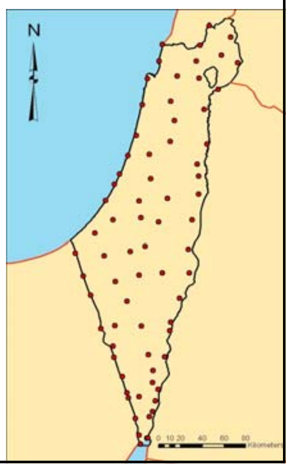
Integration of the EGM07 geoid with the local GPS-BM points

- To test the co-kriging approach a subset of the data with 72 points was selected.
- Another subset with 33 points was used as an independent verification data.

Root mean squared error of the different models

	Kriging of GPS-BM data	Kriging of EGM07 data	Cokriging GPS-BM and EGM07
Evaluation on independent data	0.20	0.31	0.15

- The augmented dataset shows a clear advantage in terms of accuracy over the two original datasets.



Summary and Conclusions

- The official Israeli geoid undulation model is a pure geometric geoid based on GPS measurements along precise leveling loops. It is updated on a regular basis using additional measurements.
- The accuracy of this model decreases with the distance from the benchmarks. Although most of the mapping and engineering projects may not require more accurate geoid model, the Survey of Israel seeks for ways to improve the official geoid model.
- The co-kriging approach was used as an inexpensive mean of enhancing the local Israeli geometric geoid model using a global gravimetric geoid.
- The augmented dataset shows a clear advantage in terms of accuracy over the two original datasets.

Summary and Conclusions

- This approach can be a great advantage for developing countries, especially where the vertical orthometric control is spars.
- Also the developed and rich countries can benefit using this approach before computing the "perfect" geoid: The best, is the enemy of the good.

SURVEY OF ISRAEL

Association of Licensed Surveyors in Israel