Application of Wavelet Analysis to GPS Stations Coordinate Time Series

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Key words: GPS, Station coordinates, Time series analysis, Discrete wavelet transform

SUMMARY

The aim of this paper consists in applying the wavelet transform into the analysis of the coordinate time series of GPS stations, in order to assess the "noise" which allows to investigate the stability of the stations and on those the "signal" which allows to determine the systematic signals such as trends and seasonal components.
In this application, we have used a set of weekly solutions of coordinate residuals of 16 well distributed stations, provided by CODE Analysis Centre of the IGS using the BERNESE Software and referred to ITRF2000. The obtained results show that the wavelet transform has well extracted the nonlinear trends and the seasonal signals contained in the studied time series such as annual and semi-annual signals. The seasonal variations are largest in the Vertical component compared to the horizontal components (North and East). The noise estimate, based on the thresholding of the wavelet coefficients, shows that the horizontal components are more stable than the Vertical component. Indeed, the standard deviation of the noise according to the North, East and Vertical components ranges between 1-3 mm, 1-2 mm and 2-5 mm, respectively.

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1. INTRODUCTION

The developments of space geodesy allowed the establishment of world geodetic networks observing constellations of satellites permanently. GPS (Global Positioning System) is one of these permanent systems, as Doppler Orbitography and Radio-positioning Integrated by Satellite (DORIS) and Satellite Laser Ranging (SLR). Great numbers of the measurements collected by these systems permit today to represent the displacement of the ground stations in terms of coordinate time series.
Time series analysis is a quite recent research field in space geodesy used in order to better apprehend the temporal variability of the physical phenomena (deformations of the earth's crust, mass transfers, geodynamic local phenomena, etc.). The most recent studies are interested particularly in the signal noise separation (denoising) of the coordinates time series based on statistical tools already used in other fields. Pioneering studies were undertaken in the late 1990s in the GPS system by King et al. (1995), Langbein and Johnson (1997), Zhang et al. (1997), Mao et al. (1999) and Williams et al. (2004). This paper is a contribution to these methodological developments. It applies the wavelet transform on the weekly position time series of GPS to separate the noise of the signal, in order to provide certain information useful to later geodynamic interpretations.
The wavelet technique permits to study the signal at different resolutions to better locate the different frequencies. The wavelet transform decomposes a signal using functions (wavelets) well localized in both physical space (time) and spectral space (frequency), generated from each other by translation and dilation (Daubechies 1992; Meyer 1992; Mallat 1999), which is well suited for investigating the temporal evolution of periodic and transient signals. The wavelet analysis has influenced much research field, of which in particular, the applications for the comprehension of the geophysics process (Kumar and Foufoula-Georgiou, 1994; Morlet et al., 1982).

2. WAVELET TRANSFORM

A wavelet is a function $\psi(t)$ of L^2 (R) (L^2 (R): Continuous functions of real variable and square integrable) which satisfies the following properties:

- Zero mean: $\int_{-\infty}^{+\infty} \psi(t)dt = 0$
- Unitary norm: $\int_{-\infty}^{+\infty} |\psi(t)|^2 dt = 1$;
- centred around t = 0.

We introduce the factor of translation (or position) u and the scale factor (or dilatation) s; therefore we define a family of functions $\psi_{u,s}$ called wavelet, deriving all of the mother wavelet ψ, such as:

$$\psi_{u,s}(t) = \frac{1}{\sqrt{s}}\psi(\frac{t-u}{s}) \quad ,s \in R;\ s \neq 0$$

The Wavelet Transform can be defined as the projection of the signal X(t) on the basis of wavelet function $\psi_{u,s}$:

$$WT(u,s) = < X, \Psi_{u,s} > = \int_{-\infty}^{+\infty} X(t)\,\overline{\psi}_{u,s}(t)\,dt$$

Where: u and s denote the translation and scale factor, respectively, and $\overline{\psi}_{u,s}$ denotes the complex conjugate of $\psi_{u,s}$.

The original signal can be reconstructed from its wavelet coefficients WT(u,s) with consideration that the wavelet ψ checks the admissibility condition:

$$C_\psi = \int_0^{+\infty} \frac{|FT(\psi(t))|^2}{t}\,dt < +\infty$$

Where: FT is the Fourier transform.

The inverse wavelet transform uses the standardization coefficient C_ψ is given by:

$$X(t) = \frac{1}{C_\psi}\int_{u\in\Re}\int_{s>0} WT(u,s)\ \psi_{u,s}(t)\,du\,\frac{ds}{s^2}$$

2.1 Discrete wavelet transform

The wavelet transform depends on the real u and s, which vary continuously; we speak therefore about a continuous wavelet transform. The continuous wavelets often yield a redundant decomposition (the information extracted from a given scale band slightly overlaps that extracted from neighbour scales). In order to avoid the redundancy, the dilatation s and the translation u take only discrete values.

$S = s_o^m$

$u = nu_o s = nu_o s_o^m$ with $n, m \in Z$; $s_0 > 1$ and $u_0 > 0$

Thus the discrete wavelet transform is formalized as follows:

$$DWT_{m,n} = < X, \Psi_{m,n} > = s_0^{-\frac{m}{2}} \int_{-\infty}^{+\infty} X(t)\,\overline{\psi}(s_0^{-m} t - nu_0)\,dt$$

To reduce redundancy, the parameters (s_o and u_o) are chosen so that the functions $\psi_{m,n}$ constitute an orthonormal basis. In general, we take a scaling factor that varies dyadic (ie powers of two): $s_o = 2$ and $u_o = 1$, and then the discrete wavelet transform becomes:

$$DWT_{m,n} = < X, \Psi_{m,n} > = 2^{-\frac{m}{2}} \int_{-\infty}^{+\infty} X(t)\,\overline{\psi}(2^{-m} t - n)\,dt$$

Here, $\psi_{m,n}$ is defined as:

$$\Psi_{m,n}(t) = 2^{-\frac{m}{2}}\,\Psi(2^{-j} t - n)$$

The Discrete Wavelet Transform (DWT) share of this consideration to lead a fast algorithm for computation the wavelet coefficients introduced by Mallat (Mallat, 1989) in the context of multi-resolution analysis.

2.2 Multi-resolution analysis

The multi-resolution analysis allows, by successive filtering, to produce a series of signals corresponding to an increasingly fine resolution of the signal. What will occur, it is that the signal is separated in two components (see fig1): one representing the approximation of the signal (represented by its low-frequency) and the other representing its details (represented by its high-frequency). To separate both, we thus need a pair of filters: a low-pass filter to obtain the approximation A, and a high-pass filter to estimate its details D. In order to not lose information, these two filters must be complementary; the frequencies cut by one must be preserved by the other. In general, the description of the signal on five different scales is sufficient to describe this signal.

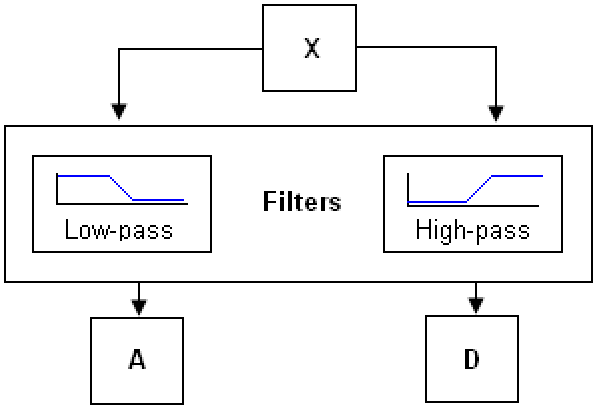

Fig. 1. The filtering process: the original signal X passes through two complementary filters and emerges as two signals (Approximation A and Detail D).

2.3 Denoising steps

The majority of wavelet algorithms use a decimated discrete decomposition of the signal (Donoho and Johnstone, 1994; Mallat, 1989). This decomposition has the characteristic to be orthogonal and to concentrate information in some great wavelet coefficients. The denoising idea is to conserve only the greatest coefficients and put the others (corresponding to the noise) at zero before reconstruction of the signal.
Assume that the observed data vector $X=[X_1,X_2,...X_N]^T$ is given by :

$$X_t = S_t + R_t\ ,\ t=1,2,...N$$

Where S_t is the true signal and R_t is Gaussian white noise centred independent and identically distributed (iid) of variance σ^2, such as $R_t \overset{iid}{\sim} N(0,\sigma^2)$.

Let $W(.)$ and $W^{-1}(.)$ denote the forward and inverse discrete wavelet transform operators. Let $D(., \lambda)$ denote the denoising operator with threshold λ.
The denoising procedure proceeds in three steps:
- Decomposition: choose a wavelet and choose a level l. Compute the wavelet decomposition of the signal X at level l; W=W(X).
- Thresholding of the wavelet coefficients; $Z=D(W,\lambda)$.
- Reconstruction: from the coefficients thresholding, one reconstruct the signal; $S=W^{-1}(Z)$.

2.4 Strategies of thresholding

Donoho and Johnstone propose two types of thresholding functions noted T_λ (Donoho and Johnstone, 1994; Donoho, 1995):

- Hard thresholding : $T_\lambda^{Hard}(W)=\begin{cases} W \text{ if } |W| > \lambda \\ 0 \text{ if } |W| \leq \lambda \end{cases}$
- Soft thresholding : $T_\lambda^{Soft}(W)=\begin{cases} W-\lambda \text{ if } W \geq \lambda \\ W+\lambda \text{ if } W \leq -\lambda \\ 0 \quad \text{ if } |W| \leq \lambda \end{cases}$

Where: W are wavelet coefficients and λ is threshold value (λ>0).

2.5 Determination of the threshold

The thresholding method suggested in this work is the VisuShrink (Donoho and Johnstone, 1994) based on the mean square error minimization. This method proposes a universal threshold λ which depends only on the number of measurements N and the noise variance σ^2. It is used when the noise is independent and identically distributed according to a centred normal law (white noise), its value is given by:

$$\lambda = \hat{\sigma}\sqrt{2\log(N)}$$

The noise variance σ^2 can be calculated by a robust estimator from the median Med of the absolute values of the wavelet coefficients at the finest scale (first level of decomposition) as follows:

$$\hat{\sigma} = \frac{Med}{0.6745}$$

Factor 0.6745 is selected from a calibration with Gaussian distribution.

3. DATA USED

For this study, we have used a set of weekly position residuals time series of 16 GPS stations (see table 1) available on the site http://maestro.obs-azur.fr/gemini/donnees/sys_ref/sys_ref.php. These data are provided by AIUB Analysis centre "CODE" of the IGS using the BERNESE Software and well distributed. The referencing time series of positions to ITRF2000 (Altamimi et al., 2002) is done by applying a

seven-parameter transformation to weekly solutions with CATREF Software. These time series are expressed in the local geodetic reference frame (dN: North component, dE: East component and dH: Vertical component).
dN, dE and dH represent the corrective terms, obtained by the least squares adjustment of the local coordinates (E, N, H) which must be added to the a priori coordinates (initial: E_0, N_0, H_0) to obtain the estimated coordinates $(\hat{E}, \hat{N}, \hat{H})$ such as:

$\hat{E} = E_0 + dE, \hat{N} = N_0 + dN$ and $\hat{H} = H_0 + dH$

Acronym	Site	Country	Lat, deg	Long, deg	Data span
EISL	Easter Island	Chile	-27.02	-109.38	1996.1-2003.8
FAIR	Fairbanks	United-states	64.97	-147.52	1996.1-2006
GODE	Greenblet	United-states	38.9	-76.8	1996.1-2006
HRAO	Hartebeesthoek	South Africa	-25.88	27.70	1996.8-2006
METS	Metsahovi	Finland	60.2	24.7	1996.1-2006
NKLG	Libreville	Gabon	0.3	9.7	2000.3-2006
NOUM	Noumea	France	-22.3	166.4	1998.2-2006
NYA1	Ny-Alesund	Norway	78.9	11.9	1998.3-2006
REYK	Reykjavik	Island	64.2	-22.0	1996.1-2006
RIOG	Rio Grande	Argentina	-53.8	-67.8	1997.7-2006
SANT	Santiago	Chile	-33.2	-70.7	1996.1-2006
STJO	St John's	Canada	47.6	-52.7	1996.1-2006
SYOG	Syowa	Antarctica	-69.00	39.58	1999.4-2006
THTI	Papeete	France	-17.58	-149.62	1998.5-2006
TLSE	Toulouse	France	43.6	1.0	2001.2-2006
USNO	Goldstone	United-states	35.3	-116.8	1997.4-2006

Table 1 Selected stations in this study

4. RESULTS AND DISCUSSION

The Multi-resolution analysis can detect the overall trend and the seasonal signals contained in the studied time series. An approximation contains the general trend (long-term evolution of the signal) of the original signal, while a detail identifies the periodic signals contained in the original signal. The figure 2 shows the trend and the annual signal contained in North (dN), East (dE) and vertical (dH) components of stations at high latitude using discrete Meyer wavelet which is very appropriate to our signal. The trends (linear or nonlinear) are identified by the approximation signal at levels 7 (Approximation 7), and the annual signals are identified by the detail signal at level 5 (detail 5). However, the figure 3 reveals the semi annual signal contained in the three components (North, East and Vertical) represented by the signal of the detail 4. The annual and semi annual signals have the largest amplitude in the Vertical component which is in the rage of 3-7 mm (see table 2) compared to 1-3 mm in the horizontal components (North and East). Globally, the horizontal displacements (trend) of the stations are typically due to plate tectonics and the Vertical displacements may be due to local subsidence or postglacial rebound (Willis et al., 2009). However, the annual and semi annual signals are due to hydrological and atmospheric loading (Dong et al., 2002; Williams and Willis, 2006).

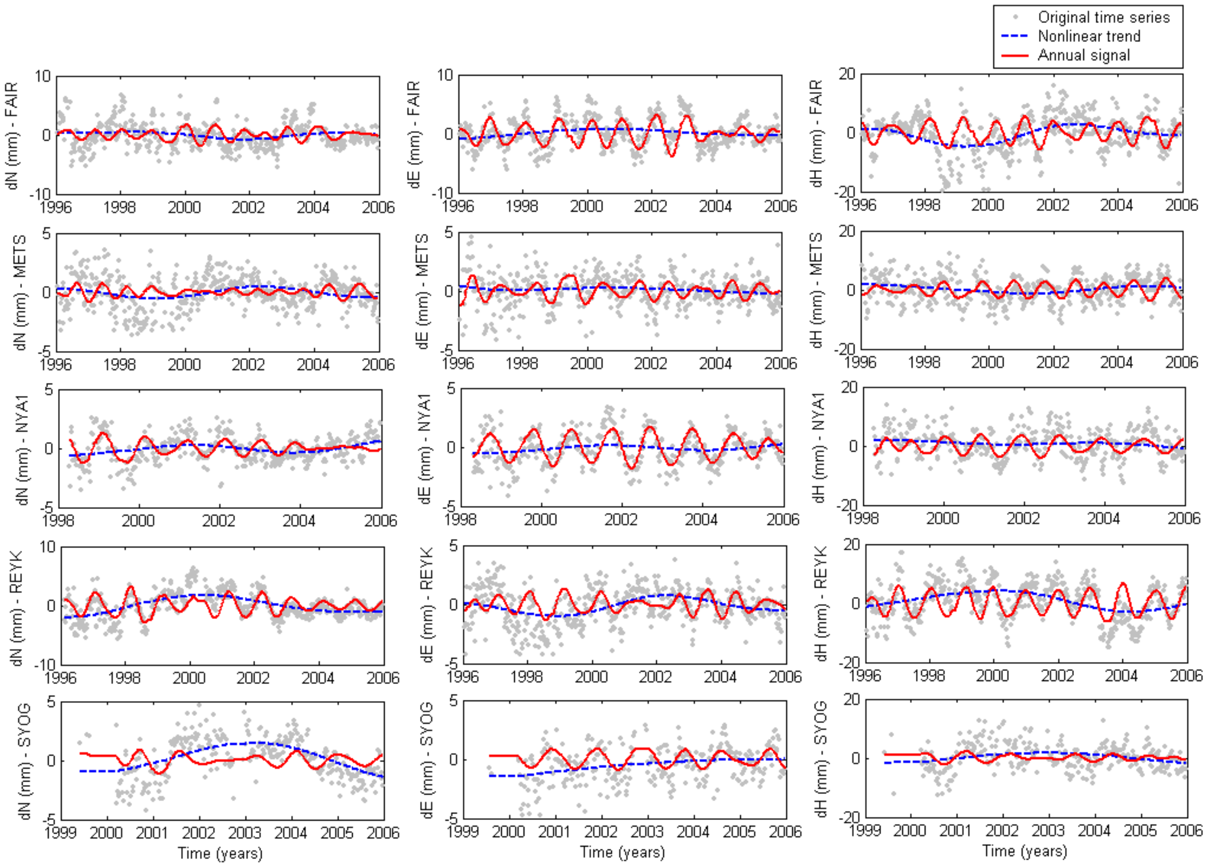

Fig. 2. Superposition of the original components (North, East and Vertical) of stations at high latitude with their nonlinear trend (Approximations 7) and annual signals (details 5) using discrete Meyer wavelet.

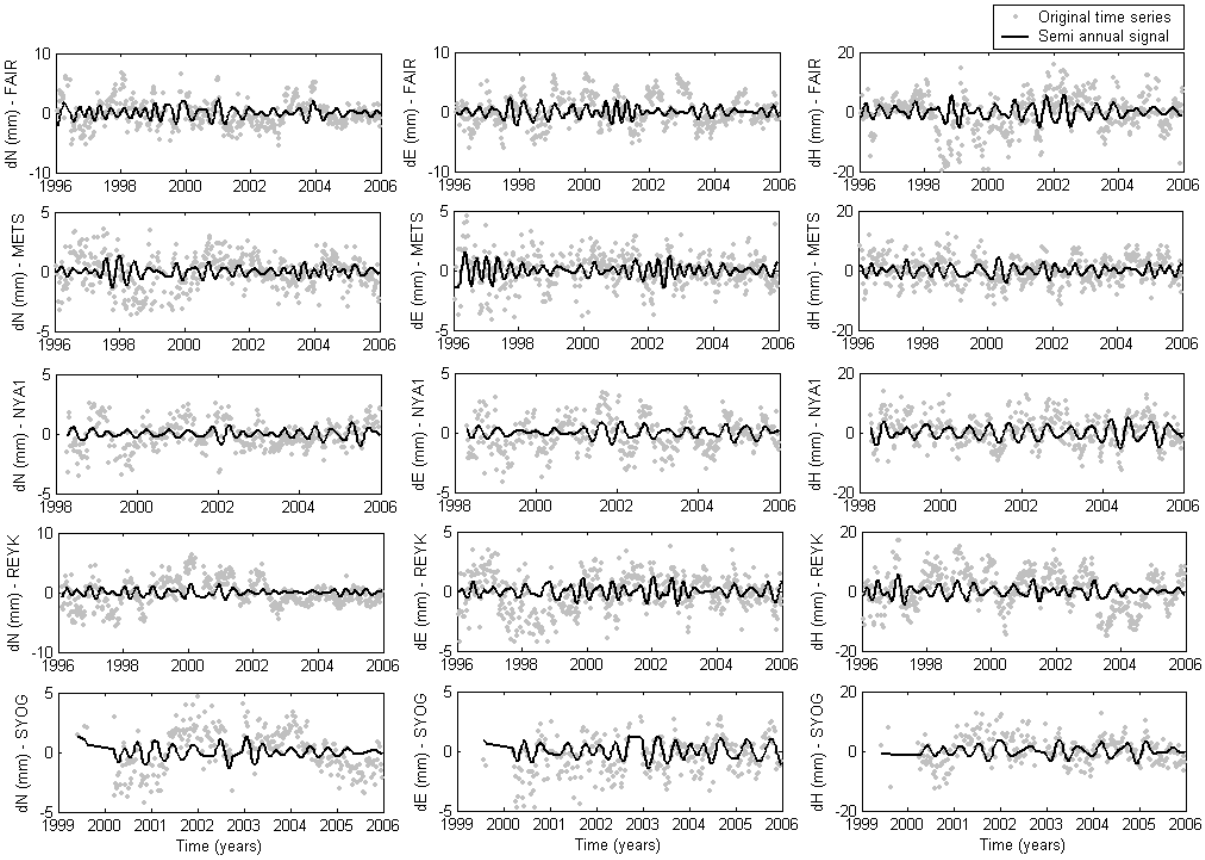

Fig. 3. Superposition of the original components (North, East and Vertical) of stations at high latitude with their semi annual signals (details 4) using discrete Meyer wavelet.

Station	North (mm)	East (mm)	Vertical (mm)

	Annual signal	Semi annual signal	Annual signal	Semi annual signal	Annual signal	Semi annual signal
EISL	4.2	4.5	1.5	2.7	8.9	7.1
FAIR	1.8	2.3	3.2	2.4	5.6	4.9
GODE	1.9	1.7	2.4	1.1	4.6	4.9
HRAO	3.1	2.2	1.4	2.1	5.6	4.6
METS	0.8	1.3	1.4	1.5	4.0	4.8
NKLG	1.2	1.9	1.3	1.8	4.7	4.1
NOUM	3.3	3.7	0.8	2.1	8.5	5.3
NYA1	1.3	1.0	1.7	1.0	3.9	5.2
REYK	3.2	1.5	1.3	1.2	6.9	4.5
RIOG	2.1	2.4	2.3	2.1	4.5	5.9
SANT	3.3	2.7	2.6	3.1	6.4	6.7
STJO	1.4	1.7	1.6	2.0	3.8	3.8
SYOG	0.9	1.4	0.9	1.3	2.3	3.9
THTI	3.5	2.5	1.6	1.4	3.8	4.6
TLSE	0.6	0.9	1.9	1.1	2.4	3.1
USNO	2.1	1.2	1.7	1.8	3.3	4.5

Table 2 Amplitudes of annual and semi-annual signals in North, East and Vertical components of studied stations.

The figure 4 represents the noise and the denoised components (dN, dE and dH) of stations at high latitude using the VisuShrink method with the soft thresholding. The wavelet coefficients used in this analysis are calculated from a decomposition of the signal at level 4 using discrete Meyer wavelet. It shows that the denoised time series are regular which is related to the continuous character of soft thresholding function.

The table 3 gives the noise statistics of the components North, East and Vertical of the studied stations. It shows that the standard deviation (STD) of the coordinate residuals noise in the horizontal (North and East) and the Vertical components ranges between 1-3 mm and 2-5 mm, respectively. As the average noise is about 0mm, the noise STD reflects the noise level. The noise level in the Vertical direction is more important compared to the horizontal one.

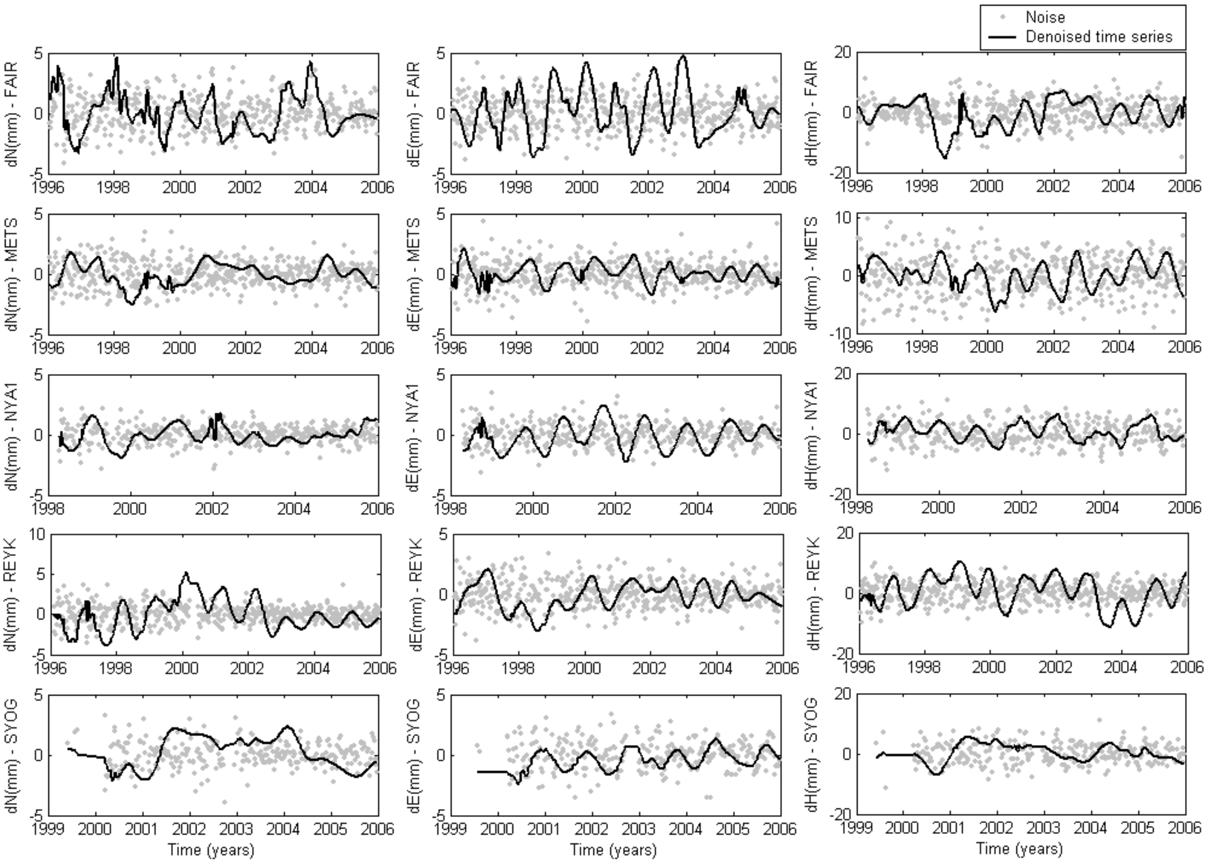

Fig. 4. Noise and denoised time series of the components (North, East and Vertical) of stations at high latitude.

Station	North [mm]			East[mm]			Vertical[mm]		
	MIN	MAX	STD	MIN	MAX	STD	MIN	MAX	STD
EISL	-9.2	10.0	2.9	-5.1	5.5	1.8	-17.7	17.2	6.1
FAIR	-4.1	3.7	1.3	-3.9	4.2	1.3	-15.2	10.9	3.8
GODE	-3.2	4.2	1.1	-3.9	3.9	1.1	-9.0	12.3	3.2
HRAO	-5.2	6.0	1.7	-4.1	4.7	1.3	-14.6	10.5	3.5
METS	-2.7	3.5	1.0	-4.0	4.3	1.1	-9.0	11.4	3.4
NKLG	-4.3	3.3	1.3	-2.8	2.1	0.9	-16.8	8.2	3.3
NOUM	-6.7	8.0	2.1	-5.0	5.3	1.4	-12.1	12.4	4.1
NYA1	-2.9	2.2	0.8	-3.2	3.5	0.9	-12.3	10.9	3.8
REYK	-3.6	3.7	1.1	-3.8	3.4	1.1	-9.8	9.8	3.2
RIOG	-4.4	4.9	1.6	-5.4	6.8	1.7	-13.7	14.2	4.0
SANT	-5.7	5.4	1.8	-5.2	4.5	1.5	-14.0	10.6	3.5
STJO	-4.1	4.5	1.3	-4.1	4.4	1.1	-12.7	8.1	2.9
THTI	-7.7	8.9	2.2	-4.5	4.1	1.2	-11.7	17.9	4.3
USNO	-3.2	3.1	1.0	-3.2	2.3	1.0	-7.7	10.8	3.1
TLSE	-1.9	1.9	0.6	-1.8	2.0	0.6	-4.8	5.6	2.1
SYOG	-4.0	3.3	1.1	-3.6	3.3	1.2	-11.3	10.8	3.5

Table 3 Noise statistics of the components (North, East and Vertical) of studied stations.

5. CONCLUSION

The main purpose of this paper is to apply the wavelet transform into the analysis of GPS coordinate time series, in order to separate the signal from the noise which allows to investigate the stability of the stations and their sites.
The data used in this analysis are the weekly time series of coordinate residuals of 16 GPS stations, computed by AIUB analysis centre "CODE" of the IGS, using the BERNESE software, referred to ITRF2000 and expressed in the local geodetic reference frame.
The application of the wavelet transform on these data permits to better assess their trend (linear and nonlinear) and their seasonal components such as annual and semi annual signals. Using the VisuShrink thresholding, based on the minimization of the noise standard deviation, the obtained results show that the noise level is higher in the Vertical component; it is in the range of 2-5 mm compared to the horizontal components (North and East) which is in the range of 1-3 mm.

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