Centre des Techniques
Spatiales

Robustness Analysis of the GPS network of Oran city, Algeria (7542)

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FIG Working Week
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0 Presentation Plan

1. **Problematic and & objectives.**
2. **Analysis of 3D geodetic networks (GPS).**
3. **Robustness analysis of GPS networks.**
4. **Program realized : Robana_3DNet.**
5. **Application: GPS network of Oran City (2009).**
6. **Conclusion and perspectives.**

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1 Problematic & objectives

- Conventionally, 3D geodetic networks are established by the union of horizontal and vertical networks.
- From decade, GPS networks have become more and more important (reference networks, topographic surveys, surveillance, auscultation, ..)
- Least Squares (LS): no robust method / Does not provide any information on the robustness of networks.
- Usual statistical analysis: evaluation of random errors! / Systematic errors or biases <values and effects / network>?
  - *Assessment of the reliability of networks approach (Theory of Baarda).*
  - *Approach to quantifying the deformation potential of networks (Concept of robustness).*

Objective: Complete analysis of 3D geodetic networks (GPS) in terms of quality, **reliability** and **robustness.** / realization of a program (ROBANA_3DNET).

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2 Analysis of GPS networks

1. Adjustment of GPS network:

Obs. relations of GPS Baseline

$$\begin{cases} v_x = \Delta X^0 - \Delta X^m + dX_v - dX_s \\ v_y = \Delta Y^0 - \Delta Y^m + dY_v - dY_s \\ v_z = \Delta Z^0 - \Delta Z^m + dZ_v - dZ_s \end{cases}$$

Matrix writing :

$$V = D.X + B$$

$D: (0\,0\,1^i\,0 \ldots -1^i\,0 \ldots 0)$

Least Squares Solution : $\sum_{i=0}^{m} p_i\, v_i^2$ Minimum.

✓**Parameters:** $\hat{X} = N^{-1}K = -(D^T.P.D)^{-1}D^T.P.B$

✓**Residuals:** $\hat{V} = D.\hat{X} + B$

✓**A posteriori variance factor :**

$$\hat{\sigma}_0^{\,2} = \frac{\hat{V}^T.P.\hat{V}}{(m-n)} = \frac{\sum_{i=0}^{m} p_i v_i^2}{m-n}$$

Weight of Obs.

$$P = \begin{pmatrix} \frac{\sigma_0^2}{\sigma_1^2} & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \frac{\sigma_0^2}{\sigma_m^2} \end{pmatrix} \qquad p_i = \frac{\sigma_0^2}{\sigma_i^2}$$

σ_0^2: A priori variance factor

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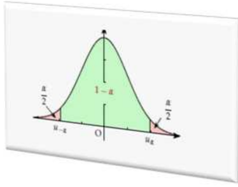
2 Analysis of GPS networks

2. Statistical Analysis :

Hypothesis : random Errors !



(a) Analysis of quality of GPS obs. :

- *Factor of a posteriori variance : **Khi-2 Test***
- *Gross errors of observations : **Student Test***

(b) Analysis of quality of estimated parameters:

- ***Precisions*** of network parameters
- ***Absolute Error Ellipsoids***
  (**2D:** error ellipsis; **1D:** error interval)

$$C_{\hat{x}} = \hat{\sigma}_0^{\,2}.N^{-1} = \hat{\sigma}_0^{\,2}.(D^T.P.D)^{-1}$$

$$C_{\hat{x}} = \begin{pmatrix} V(x_1) & cov(x_1,x_2) & \cdots & cov(x_1,x_n) \\ cov(x_2,x_1) & V(x_2) & \dots & cov(x_2,x_n) \\ \dots & . & & \dots \\ \dots & . & & \dots \\ cov(x_n,x_1) & cov(x_n,x_2) & \cdots & V(x_n) \end{pmatrix}$$

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3 Robustness Analysis of GPS networks

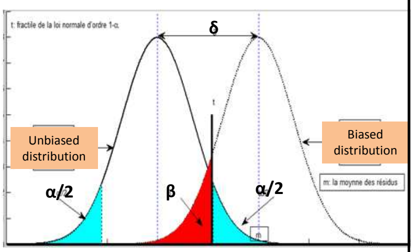
1. Reliability of GPS networks :

Network ability to detect to detect blunders and to estimate the effects that undetected blunders may have on a solution (adjusted parameters). « Baarda's theory »

Reliable network **:** minimise **non detectable errors** in obs. ⇒ minimise **effets of these errors** on adjusted parameters.

❑ **Reliability elements :**

- **Redundancy numbers** : $R = I - A(A^t PA)^{-1} A^t P$
- **Inner Reliability :** $\nabla_{li} = \sigma_{Vi}.\dfrac{\delta_0}{\sqrt{r_i}}$
- **External Reliability:** $\nabla_x = (A^t P A)^{-1} A^t P \nabla_{li}$



α : Type I Error (*prob. of rejecting a good obs*)

β : Type II Error (*prob. of accepting a blunber*)

δ : Non-centrality parameter

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3 Robustness Analysis of GPS networks

2. Robustness of GPS networks :

Combinaison of **reliability** and **deformation** of network [Vanicek et al., 1991].

➢ **Deformation ?**

- **Displacement field :**

$$\Delta X_i = \begin{bmatrix}\Delta x_i\\ \Delta y_i\\ \Delta z_i\end{bmatrix} = \begin{bmatrix}u_i\\ v_i\\ w_i\end{bmatrix} \qquad \Delta X = \nabla x = (A^T.P.A)^{-1}.A^T.P.\nabla_{bi}$$

Max External reliability

where

$$a_i + \frac{\partial u_i}{\partial x}(X_j - X_i) + \frac{\partial u_i}{\partial y}(Y_j - Y_i) + \frac{\partial u_i}{\partial z}(Z_j - Z_i) = u_j$$

$$b_i + \frac{\partial v_i}{\partial x}(X_j - X_i) + \frac{\partial v_i}{\partial y}(Y_j - Y_i) + \frac{\partial v_i}{\partial z}(Z_j - Z_i) = v_j$$

$$c_i + \frac{\partial w_i}{\partial x}(X_j - X_i) + \frac{\partial w_i}{\partial y}(Y_j - Y_i) + \frac{\partial w_i}{\partial z}(Z_j - Z_i) = w_j$$

j=1, .. t (number of liaisons)
i=1, .. n (number of points)

- **Strain tensor :**

$$E(x,y,z) = \begin{bmatrix}\frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z}\\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z}\\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z}\end{bmatrix}_{(x,y,z)} = \begin{bmatrix}\varepsilon_{ux} & \varepsilon_{uy} & \varepsilon_{uz}\\ \varepsilon_{vx} & \varepsilon_{vy} & \varepsilon_{vz}\\ \varepsilon_{wx} & \varepsilon_{wy} & \varepsilon_{wz}\end{bmatrix}_{(x,y,z)}$$

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3 Robustness Analysis of GPS networks

- **Computation of optimal displacements :**

$$d_i = \sqrt{u_i^2 + v_i^2 + w_i^2}$$

i=1, .. n (number of points)

$$\begin{bmatrix}u_i\\ v_i\\ w_i\end{bmatrix} = \begin{bmatrix}\frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z}\\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z}\\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z}\end{bmatrix}_i . \begin{bmatrix}X_i - X_0\\ Y_i - Y_0\\ Z_i - Z_0\end{bmatrix}$$

X_0, Y_0, Z_0 : initial conditions

$$\min_{(X_0,Y_0,Z_0 \in \mathbb{R})} \sum_{i=1}^{n} \|\Delta\vec{r}\|_i = \min_{(X_0,Y_0,Z_0 \in \mathbb{R})} \sum_{i=1}^{n} u_i^2 + v_i^2 + w_i^2$$

- **Displacement thresholds** : [GSD/Canada, 1996]

$$\delta_i = \sqrt{\sigma_{a_{95i}}^{\ 2} + \sigma_{b_{95i}}^{\ 2} + \sigma_{h_{95i}}^{\ 2}}$$

For probability of 95% :

$$\sigma_{a_{95i}} = 2.45.\sigma_{a_i}$$
$$\sigma_{b_{95i}} = 2.45.\sigma_{b_i}$$
$$\sigma_{h_{95i}} = 1.96.\sigma_{h_i}$$

- **Measure of robustness :**

$d_i \leqslant \delta_i$: Robust Point

$d_i > \delta_i$: Weak Point

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3 Robustness Analysis of GPS networks

FIG SOFIA 2015

3. Strain tensor 3D:

Strain tensor :

$$E(x,y,z) = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{bmatrix}_{(x,y,z)} = \begin{bmatrix} \varepsilon_{ux} & \varepsilon_{uy} & \varepsilon_{uz} \\ \varepsilon_{vx} & \varepsilon_{vy} & \varepsilon_{vz} \\ \varepsilon_{wx} & \varepsilon_{wy} & \varepsilon_{wz} \end{bmatrix}_{(x,y,z)}$$

$$E = S + A$$

$$S = \begin{bmatrix} \varepsilon_{ux} & \frac{1}{2}(\varepsilon_{uy}+\varepsilon_{vx}) & \frac{1}{2}(\varepsilon_{uz}+\varepsilon_{wx}) \\ \frac{1}{2}(\varepsilon_{uy}+\varepsilon_{vx}) & \varepsilon_{vy} & \frac{1}{2}(\varepsilon_{vz}+\varepsilon_{wy}) \\ \frac{1}{2}(\varepsilon_{uz}+\varepsilon_{wx}) & \frac{1}{2}(\varepsilon_{uz}+\varepsilon_{wx}) & \varepsilon_{vy} \end{bmatrix}$$

Symetric part

Anti-symetric part

$$A = \begin{bmatrix} 0 & -\omega_{xz} & \omega_{yz} \\ \omega_{xz} & 0 & -\omega_{xy} \\ -\omega_{yz} & \omega_{xy} & 0 \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{2}(\varepsilon_{uy}-\varepsilon_{vx}) & \frac{1}{2}(\varepsilon_{uz}-\varepsilon_{wx}) \\ \frac{1}{2}(\varepsilon_{uy}-\varepsilon_{vx}) & 0 & -\frac{1}{2}(\varepsilon_{vz}-\varepsilon_{wy}) \\ -\frac{1}{2}(\varepsilon_{uz}-\varepsilon_{wx}) & \frac{1}{2}(\varepsilon_{vz}-\varepsilon_{wy}) & 0 \end{bmatrix}$$

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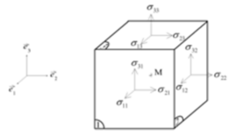
3 Robustness Analysis of GPS networks

FIG SOFIA 2015

3. Strain tensor 3D:



Primitives of Deformation :

Dilatation

Average :

$$\sigma = \frac{1}{3}(\lambda_1 + \lambda_2 + \lambda_3) = \frac{1}{3}(\varepsilon_{ux} + \varepsilon_{vy} + \varepsilon_{wz})$$

Total :

$$\lambda = \sqrt{\lambda_1^2 + \lambda_2^2 + \lambda_3^2}$$

Twist

Component :

$$\omega_{xy} = \frac{1}{2}(\varepsilon_{vz} - \varepsilon_{wy})$$

$$\omega_{xz} = \frac{1}{2}(\varepsilon_{uz} - \varepsilon_{wx})$$

$$\omega_{yz} = \frac{1}{2}(\varepsilon_{uy} - \varepsilon_{vx})$$

Total :

$$\Omega = \sqrt{\omega_{xy}^2 + \omega_{xz}^2 + \omega_{yz}^2}$$

Shear

Simple :

$$\begin{cases} \tau_{xy} = -\tau_{yx} = \frac{1}{2}(\varepsilon_{ux} - \varepsilon_{vy}) = \frac{1}{2}\left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right) \\ \tau_{xz} = -\tau_{zx} = \frac{1}{2}(\varepsilon_{ux} - \varepsilon_{wz}) = \frac{1}{2}\left(\frac{\partial u}{\partial x} - \frac{\partial w}{\partial z}\right) \\ \tau_{yz} = -\tau_{zy} = \frac{1}{2}(\varepsilon_{vz} - \varepsilon_{wy}) = \frac{1}{2}\left(\frac{\partial v}{\partial z} - \frac{\partial w}{\partial y}\right) \end{cases}$$

Pure :

$$\begin{cases} \upsilon_{xy} = -\upsilon_{yx} = \frac{1}{2}(\varepsilon_{uy} + \varepsilon_{vx}) = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \\ \upsilon_{xz} = -\upsilon_{zx} = \frac{1}{2}(\varepsilon_{uz} + \varepsilon_{wx}) = \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \\ \upsilon_{yz} = -\upsilon_{zy} = \frac{1}{2}(\varepsilon_{vy} + \varepsilon_{wz}) = \frac{1}{2}\left(\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) \end{cases}$$

Total :

$$\gamma = \sqrt{\gamma_{xy}^2 + \gamma_{xz}^2 + \gamma_{yz}^2}$$

.. For an easier interpretation of strain tensor

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3 Robustness Analysis of GPS networks

FIG SOFIA 2015

3. Strain tensor 3D:

Invariants of deformation primitives [Vanicek et al., 2001] and [Berber, 2006]:

Mean Dilatation
Robustness in Scale

$$\sigma = \frac{1}{3}(\lambda_1 + \lambda_2 + \lambda_3) = \frac{1}{3}(\varepsilon_{ux} + \varepsilon_{vy} + \varepsilon_{wz})$$

Max Shear
Robustness in Configuration

$$\gamma_{max} = \max(\lambda_i) - \min(\lambda_i),\ i = 1{,}3$$

Total Differential Rotation
Robustness in twist

$$\Omega = \sqrt{\omega_{xy}{}^2 + \omega_{xz}{}^2 + \omega_{yz}{}^2}$$

.. In this study, 03 components are choosen among different strain tensor primitives, which are deformation invariants in the 3D case.

Here the robustness is expressed in deformation.

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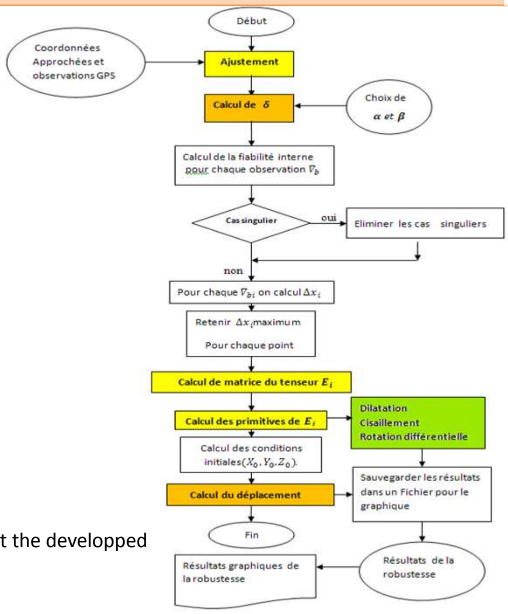
3 Robustness Analysis of GPS networks

FIG SOFIA 2015

4. Algorithm :



➢ This procedure is implemented at the developped program ***Robana_3DNet***

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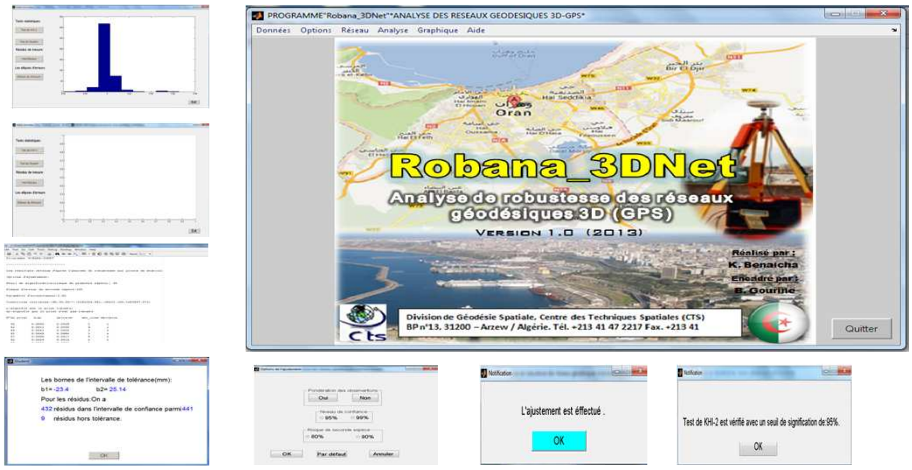
4 Realised Program : Robana_3DNet

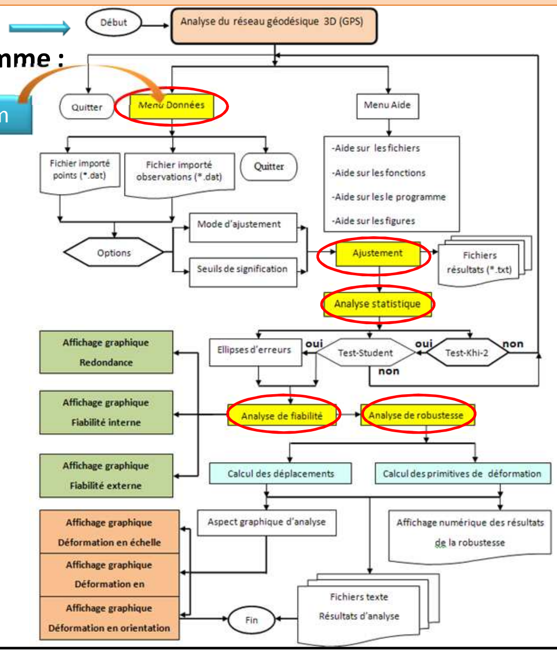


- **ROBANA_3DNET** (**ROB**ustness **ANA**lysis of **3D** geodetic **NET**works) : MATLAB program, Adjustment & Reliability and Robustness Analyses of GPS nets. [DGS /CTS, Arzew].
- **Modules** : Adjustment / Statistical Analysis / Reliability Analysis/ Robustness Analysis

4 Realised Program : Robana_3DNet

2. Organigramme :

Process/WinPrism



5 Application: GPS network of Oran City

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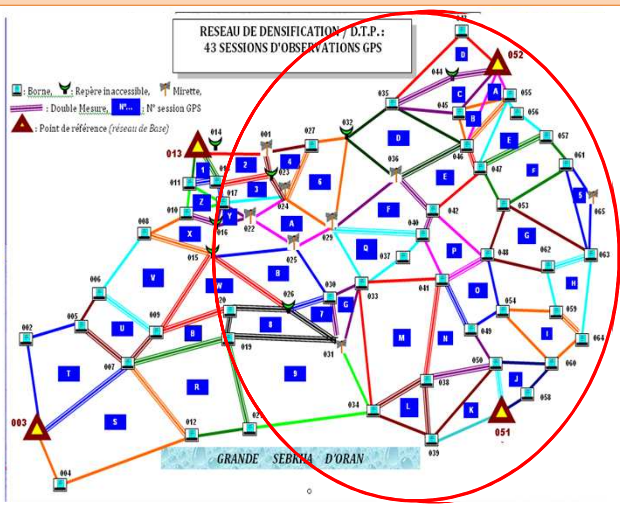
5 Application: GPS network of Oran City



WinPrism / PROCESS

- 28 sessions (45 points)
- σ GPS baselines = **± 4.3 mm** (±3.2 mm, ±1.5 mm, ±2.3mm : ΔX, ΔY and ΔZ, *resp.*)

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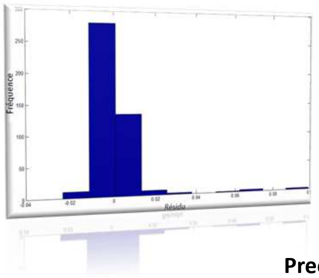
FIG SOFIA 2015

1. Adjustment :

Number of Parameters	Number of Observations	Number of Freedom Degree	Number of Fixed points	σ_0 a Priori Standard deviation	Probability Thresholds
132	147 GPS Baselines (441 components)	349	01 (point 37)	1.	$\alpha = 5\%$ $\beta = 20\%$



$\hat{\sigma}_0$ a Posterior Standard deviation	Khi-2 Test	Student Test
0.167	positive	7 baselines suspected

Precisions

	σ_E(m)	σ_N (m)	σ_U (m)
Average	0.008	0.004	0.009
RMS	0.002	0.001	0.002
Minimum	0.005	0.003	0.006
Maximum	0.013	0.007	0.014

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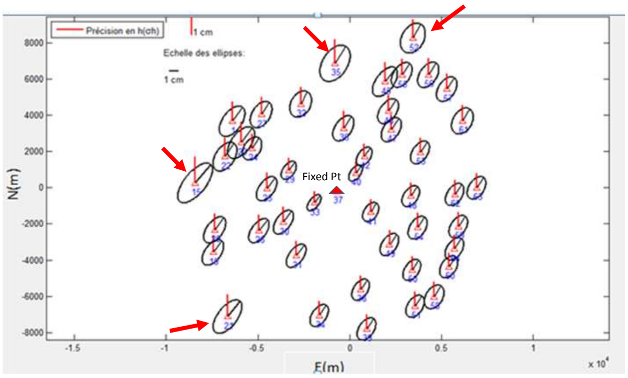
FIG SOFIA 2015

1. Ajustement :

- Error Domains

	a (m) major semi-axis	b (m) minor semi-axis
Average	0.008	0.004
RMS	0.002	0.001
Minimum	0.005 (pts: 33,40)	0.003 (pts:26,40,42,49
Maximum	0.013 (pts : 15)	0.007 (pts: 35)



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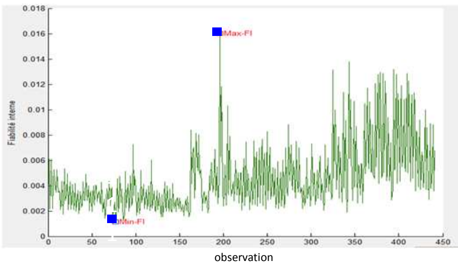
2. Analyse de fiabilité:

- Redundancy numbers

Measure of absorption of a blunder:
indication of reliabilty of obs.

- Inner Reliablity (FI)

	Fi(ΔX)	Fi(ΔY)	Fi(ΔZ)
Average	0.006	0.003	0.004
RMS	0.003	0.001	0.003
Minimum	0.002	0.001	0.002
Maximum	0.016	0.010	0.012



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5 Application: GPS network of Oran City

- External Reliablity (FE)

	Fe(dE)	Fe(dN)	Fe(dU)
Average	-0.004	-0.001	-0.001
RMS	0.004	0.002	0.003
Minimum	-0.015	-0.006	-0.008
Maximum	0.006	0.004	0.007

..in m

- External relibality of network is of about few mm→ reliable network.
- However, max values ≤ 15 mm have important dimension of error ellipses.

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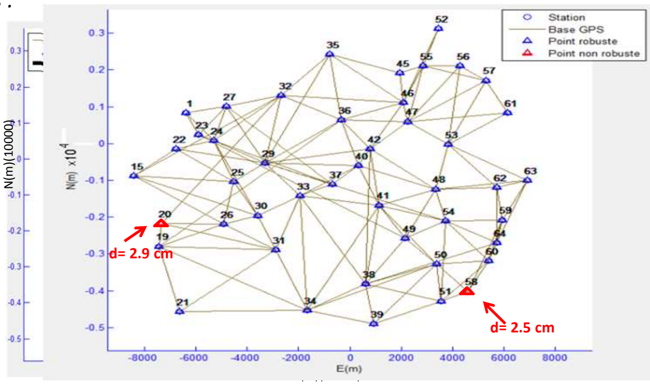
2. Analyse de robustesse :

Network is robust ?

	Disp.(d) (m)	Treshold (δ) (m)
Average	0.003	0.024
RMS	0.007	0.060
Minimum	0.0	0.0
Maximum	0.030	0.040

d: displacement

In terms of displacements :



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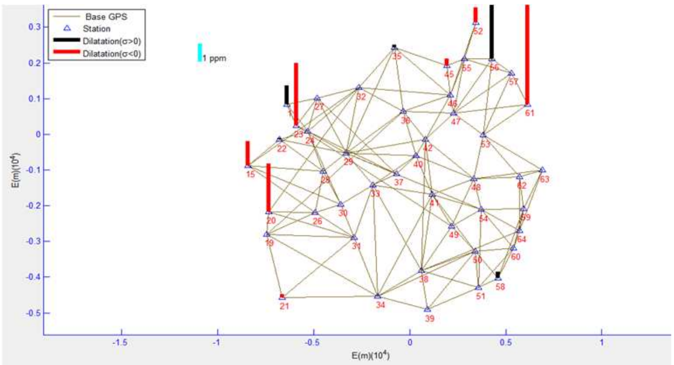
2. Analysis of robustness :

Network is robust ?

	Dil (σ) (ppm)	Shear-max (λ) (ppm)	Rot (Ω) (ppm)
Average	-9.0	61.0	27.0
Minimum	0.0	0.0	0.0
Maximum	400.0	4200.0	3200.0

In terms of deformations..

Robustness in scale (dilatation)



ppm: part per million

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6 Conclusion and perspectives

- ✓ Reliability Analysis of aspects and robustness of 3D geodetic networks (GPS) are treated.
- ✓ **Valorization of the work**: realization of **Robana_3DNet** program (in MATLAB), at (DGS / CTS) → [Adjustment, Statistical Analysis, reliability and robustness analyses: GPS networks].
- ✓ **Validation of the program**: test network (45 points) of the GPS network of the city of Oran (2009).
- ✓ **Results:** σ GPS baselines ± 4 mm (PROCESS / WinPrism); σ GPS points ± 9 mm in position and height. Error Domains (ellipses and ranges of errors) are more important in points of network edge.
- ✓ **Network reliability:** redundancy [15%, 98%]; bias (internal reliability) ~ 8 mm; external network reliability 4 mm in horizontally and 1 mm in altimetry → reliable network.
- ✓ **Network Robustness:** Robust network throughout except few pts on perimeter where displacements and deformations are significant due to low number of connections and to configuration.
- ✓ **Design of GPS networks :** criteria of reliability and robustness.

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6 Conclusion and perspectives

Outlook:

To enrich ROBANA_3DNET program, it is necessary to:

- ➢ Validate the program on other GPS networks, including large networks.
- ➢ Develop an automatic diagnosis of the network analysis results.
- ➢ Integrate a module of significance of primitive strain tensor, based on Monte Carlo method.

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Thank you for your attention

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