

## **RG 2000 – the new gravity reference frame of Sweden**

**Andreas ENGFELDT, Martin LIDBERG, Sweden, Marcin SEKOWSKI, Przemyslaw DYKOWSKI, Jan KRYNSKI, Poland, Jonas ÅGREN, Per-Anders OLSSON, Henrik BRYSKHE, Holger STEFFEN, Sweden, Jens Emil NIELSEN, Denmark**

**Key words:** Gravity, Reference Frame, absolute gravimetry, FG5, A10, RG 2000

### **SUMMARY**

We present the new gravity reference frame RG 2000 for Sweden, including its realization through a combination of absolute gravity observations and a network of relative observations. The main motivation for the work is the increased need for improved geoid models from GNSS height determination, which calls for additional gravity observations and quality assurance of existing data. In this perspective, a new modern gravity system and the renovation of the high order gravity network is considered as a moderate strategic investment which provides a firm foundation for further activities.

The previous Swedish gravity reference frame, RG 82, was based on four absolute gravity observations in Scandinavia in 1976 by the Italian absolute gravimeter IMGIC. Although the gravity level of this system in land uplift epoch 1982.0 agrees surprisingly well to RG 2000 (some 30  $\mu\text{Gal}$  difference), a considerable improvement is possible with modern absolute instruments. The ongoing glacial isostatic adjustment in Fennoscandia influences all geodetic reference systems over time and make the epoch essential. The epoch of RG 2000 is 2000.0, which corresponds well with the epochs of the national height system, RH 2000 and the national 3D system, SWEREF 99.

In the autumn of 2006 Lantmäteriet in Sweden purchased the FG5-233 absolute gravimeter and has since the spring of 2007 observed the absolute gravity at 13 sites in Sweden with the highest possible accuracy to date. The scientific purpose is to study the gravity change due to glacial isostatic adjustment. However, these stations will also form a firm base for the new gravity system and its realization.

The RG 2000 project started in 2011 with the first field campaign using the portable A10-020 absolute gravimeter, owned by the Institute of Geodesy and Cartography (IGiK), Poland. During totally five field campaigns from 2011 to 2015, 95 points were observed with that instrument. Almost half of the points were previously used in RG 82 or the even older RG 62, which means that good connections between the systems have been established. In addition, two points were observed with the A10-019 absolute gravimeter owned by DTU Space in Denmark. All the still available points in the first two orders of the RG 82 gravity network were added to the new network by using relative observations from 1975 to 2002 and new observations from 2013 to 2017 to connect them to the FG5 and new A10 points. Epoch reduction of the gravity observations was done using the land uplift model NKG2016LU, which

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is a product of the joint work of the Nordic Geodetic Commission (NKG) and a linear relation between land uplift and gravity change. The adjustment was performed with the software Gad (Gravity adjustment), developed in-house by Lantmäteriet. Finally, it should be mentioned that RG 2000 is a zero permanent tide system.

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## 1. INTRODUCTION

There has been a tremendous development in surveying engineering over the last decades, where Network RTK has become available in practically all European countries, and is nowadays a standard tool for the surveyors. While the uncertainties from densified Network RTK networks for construction work are approaching the sub-centimeter level, also in the vertical (above the ellipsoid), this high accuracy may easily get lost while converting the GNSS heights to “gravity related heights” in the national height frame using a geoid model. Therefore, surveyors are constantly asking for “better geoid models”.

Thanks to the recent dedicated satellite gravity field missions (CHAMP, GRACE and GOCE), the improvements in global geopotential models is on the same level as the developments in GNSS, with an uncertainty at the 1 cm level for a resolution of about 100 km. However, for precise geoid models that surveyors are asking for, we need accurate terrestrial gravity observations with much higher spatial resolution (typical 3-5 km spacing).

While developing the strategic plan for Geodetic infrastructure in Sweden in 2010 (Lantmäteriet 2010), it was thus concluded that gravity observations will be a major task for the years to come. In that perspective, it was also decided to establish a new gravity reference network, and to develop a new national reference frame for gravity.

Due to its location in to the Fennoscandian postglacial rebound (PGR) area, Sweden is subject to crustal deformations with a maximum land uplift of about 1 cm/yr. In precise geodetic work, the epoch of observations and epoch of geodetic reference frames are therefore of outmost importance. Thus, it was decided to name the new gravity frame RG 2000, with land uplift epoch 2000, to be compliant to the national reference frames RH 2000 and SWEREF 99 in height and 3D, respectively (Kempe et al 2016).

## 2. BACKGROUND

There are currently two gravity systems in use in Sweden, see Figure 1. The RG 82 system was established in 1981-1982 with the use of two LaCoste & Romberg model G gravimeters and is based on four absolute gravity observations (two in Sweden, one in Finland and one in Denmark) by the Italian instrument IMGIC in 1976 (Haller & Ekman 1988). The absolute level of RG 82 has lately proved to be much better than expected, with a bias of about 30  $\mu$ Gal compared to modern absolute observations, if the land uplift is accounted for (see below).

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The First Order Network in RG 82 is not a network in any real meaning, since only 15 points have been measured from more than one starting point. However, it is still a densification of the Zero Order Network and consists of 149 points and was finished in October 2002 (Engfeldt 2016a).

The old RG 62 system was established between 1960-1966 with the use of a Worden Master gravimeter and was connected to Potsdam via the European Calibration System (ECS) 1962 (Pettersson 1967). Due to the well-known bias in the determined absolute level of Potsdam in the ECS 62, and due to poor relative instruments, it is separated from RG 2000 with about 14,6 mGal.

The distances between the points in both RG 82 and RG 62 were in general around 40-60 kilometres. Despite RG 62 in general covered the area of Sweden, there were a lot of big gaps in the coverage, where the nearest site was more than 100 km away. Of the 185 points, only 23 were marked with a benchmark, so they could be identified. Many of the points are situated on church steps, but unfortunately for very few of them the place on the step is precisely described.

About 75% of the terrestrial gravity observations used as a basis for geoid determination are originally measured relative to the RG 62 network. It is therefore reasonable to consider also RG 62 while establishing a new reference frame for gravity, and if possible find common points to facilitate the development of improved transformations between the existing RG 62 and RG 82, and the new RG 2000 gravity system.

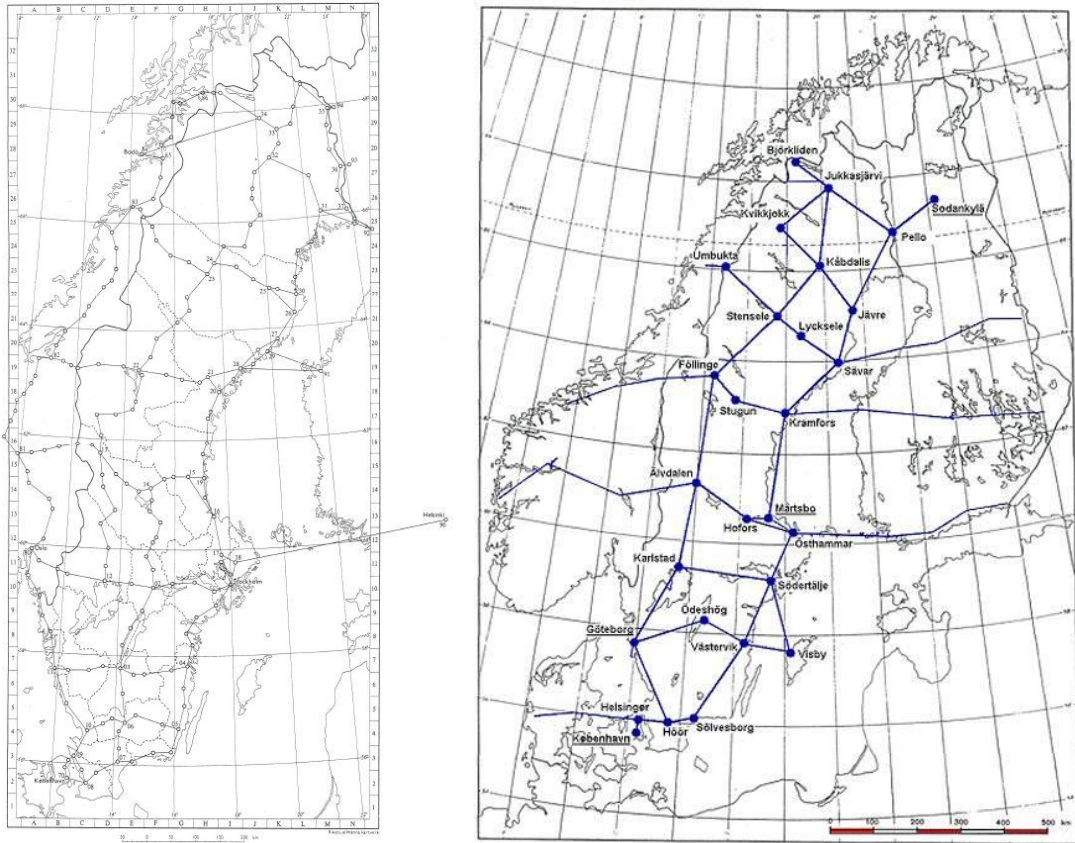
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**Figure 1.** *Left:* The RG 62 Network, from Pettersson (1967). *Right:* The Zero Order Network of RG 82, from Haller & Ekman (1988).

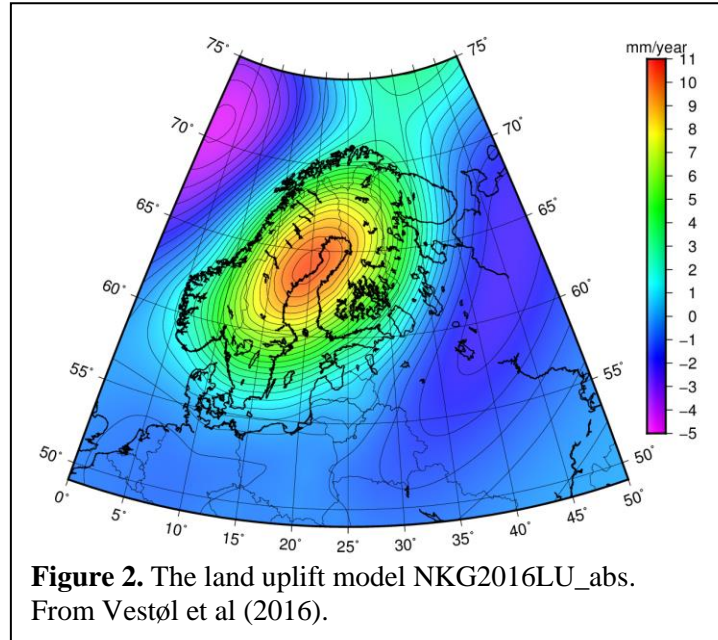
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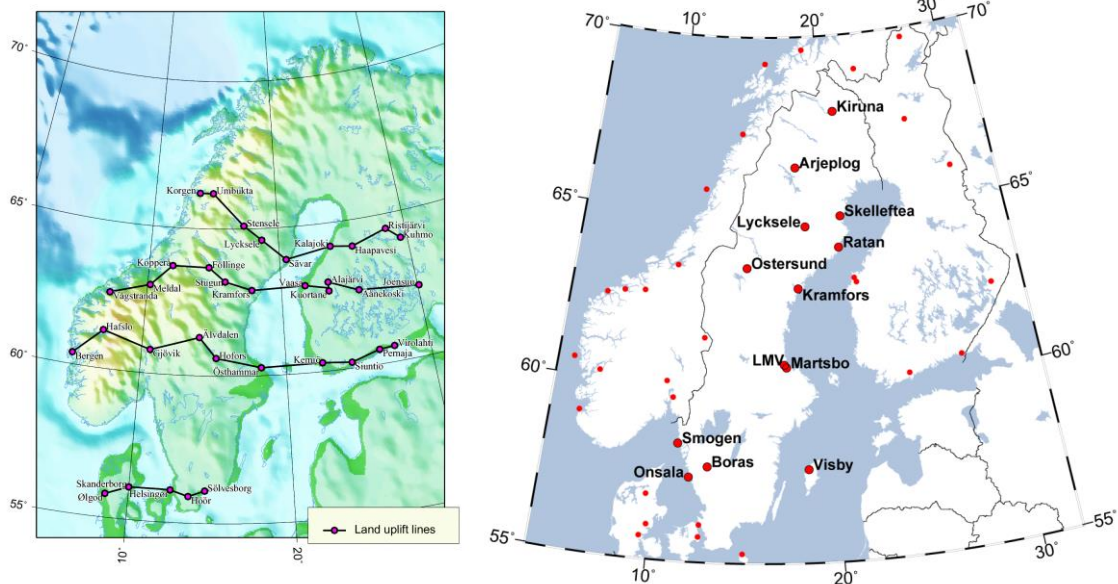
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## 2.1 Land uplift research

The Fennoscandian Post Glacial Rebound (Figure 2) has been a subject of scientific interest since the 17<sup>th</sup> century (Ekman 2009). From the perspective of gravity observations, it started in the 1960's when the Fennoscandian land uplift gravity lines were established. The purpose was to better understand the PGR process by determining the relation between gravity change and geometric land uplift ( $\dot{g}/\dot{h}$ ) from observations (The relation would help identifying to what extent the PGR is an elastic phenomenon, or if there is an inflow of mantle material to the PGR area.).



In total, four lines were established (Figure 3 left), where the 63 degree latitude line has been observed most times, with observations of the complete line almost every 5 years between 1966 and 2003. For these measurements, only LaCoste & Romberg D- and G-models were used. In total, about 20 instruments participated in one or more of these observation campaigns. All raw data from the measurements between 1966 and 1984 were published in Mäkinen et al (1986). The raw data from the measurements between 1985 and 2003 have not been published, but conclusions based on them can be found in Mäkinen et al (2005).



**Figure 3.** *Left:* The location of the Fennoscandian land uplift gravity lines. *Right:* The points included in the NKG Absolute Gravity project.

## 2.2 Modern absolute gravity observations

Apart from the observations in 1976 by the Italian IMGC instrument, observations in the region have also been performed in the 1990s by FGI (Finnish Geodetic Institute), with a JILAg instrument, BKG (Bundesamt für Kartografie und Geodäsie, Frankfurt, Germany) and NOAA (National Oceanic and Atmospheric Administration, Boulder, Colorado, USA) with FG5 instruments. In connection to these observations, new gravity points designed especially for absolute gravity demands were established.

In 2003, the NKG (Nordic Geodetic Commission) absolute gravity project started, and practically replaced the observations of the land uplift gravity lines (Figure 3 right). The project included all the mapping agencies in the Nordic countries, as well as IfE (Leibniz University in Hannover, which observed the most stations during the first four years of the project), NMBU (The Norwegian University of Life Sciences, Ås, Norway) and BKG. For this project, several new absolute gravity points were established in the Nordic countries, all co-located with permanent GNSS stations (Engfeldt 2016a). The first results from the absolute gravity surveys have been published in Gitlein (2009).

In October 2006 Lantmäteriet purchased an FG5 instrument (Figure 4), and from 2007 onwards it is used for regular absolute gravity observations at the stations in Sweden.

The Swedish standard procedure to measure absolute gravity is as follows:

- Two orientations, 24 hours in north orientation and 24 hours in south orientation
- 24 sets in every orientation (in 2007, 48 sets in every orientation)
- 50 drops (observations) per set
- All observations not within the 3 sigma level are regarded as outliers and are removed directly by the g-software



**Figure 4.** The Lantmäteriet FG5-233 absolute gravimeter, measuring at Smögen AA, Sweden.

In 2009, Onsala Space Observatory, purchased a superconducting gravimeter GWR-054. For the new instrument, a new gravity building was established. There is a separate room for the superconducting gravimeter and a room with three different piers for absolute gravity measurements. These new absolute points (Onsala AA, AB and AC) are replacing the old (Onsala AN and AS) and are considered the best gravity points in Sweden.

### 3. RG 2000 WORK

#### 3.1 Inventory of points from the old networks in 2011

The first step in the RG 2000 work was to investigate which points from the old networks RG 82 and RG 62 still existed and could be potentially used in RG 2000. Of special importance was to identify points which could be used for the portable A10 absolute gravimeter, since the main idea was to cover Sweden with one A10 observation every 50-70 km. It was found that most of the RG 82 points and about half of the RG 62 still were available and usable for relative gravimeters (Engfeldt 2016a). Unfortunately, there were less points than expected which fulfilled the requirements for the A10. This meant in the end that 55 new points were established and that more relative observations than expected were required (see 3.3).

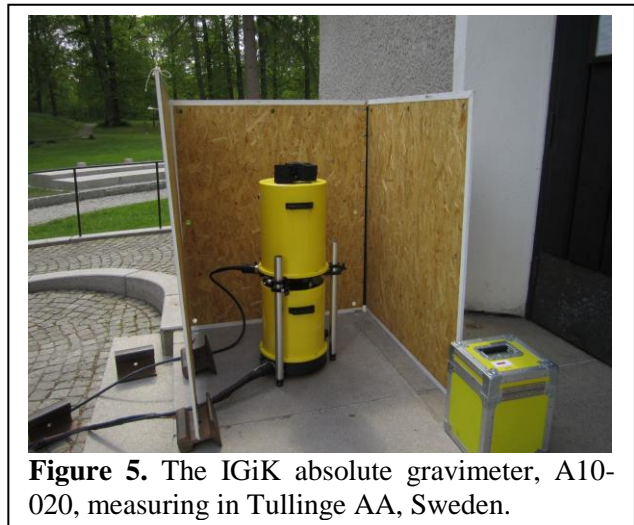
#### 3.2 A10 observations for RG 2000

In 2011, the first observations done specific for RG 2000 were performed when the A10-020 (owned by IGIK, Figure 5) visited Sweden for first observations at 12 points. This was performed as a test to see if the A10 observations were suitable for determining our old outdoor points (i.e. the RG 62- or RG 82-sites) (Engfeldt 2016a). The test observations were conducted such that at each point two setups were performed with the instrument oriented in two different directions, 120 degrees in between, since with this instrument the influence of the Coriolis Force



was tested, during A10-020 measurements in Finland, not to be significant (Makinen et al., 2010). One single setup consists of 8 sets, each set consists of 120 drops performed every second. Two setups usually took less than 2 hours per station. Typically in case the results from the two orientations differed less than 10  $\mu\text{Gal}$  they were considered satisfactory, otherwise one more orientations were performed to improve the final result.

The result from these observations was very satisfactory, so it was decided that four more campaigns with A10 should be performed, two in 2012, one in 2013 and one in 2015 (Engfeldt 2016b). In total, 98 points were visited, of which three were the FG5-points Mårtsbo AA, Onsala AA and Kiruna AA. At least one of these three points was visited once during every campaign, where it was used as reference value to check that the A10 results were reliable over time. The eight points with the largest differences between the orientations were re-measured in 2013/15 just to check if any gross errors occurred.



**Figure 5.** The IGIK absolute gravimeter, A10-020, measuring in Tullinge AA, Sweden.

In April and June of 2012, the A10-019 absolute gravimeter (owned by DTU Space), performed surveys along the 56<sup>th</sup> degree land uplift line. Unfortunately, it was not possible to measure the relative gravity points (included in RG 82) in Höör and Sölvesborg with an A10. Therefore, new points were established at Höör church and Sölvesborg church, to be connected to the old points by relative gravity observations.

### 3.3 Relative gravity observations

Before the work with RG 2000 started, we knew that we had very many good old relative gravity observations which could also be used for this network. Since we wanted a real network, the new A10 points should be connected to at least one point in the old RG 82 network or one of the FG5 points. This also meant a rough check for gross errors in the A10 observations (Engfeldt 2016b) and it should be done by relative gravity observations. At the same time, several more relative gravity observations were performed to strengthen the network. The relative gravity observations and instruments used in the calculations and adjustments are the following:

- RG 82 zero order campaign 1981-82, LaCoste & Romberg G54 and G290
- RG 82 first order campaign 1984-96, 2001-02, LaCoste & Romberg G54 and G290
- RG 2000 campaign 2015-17, LaCoste & Romberg G54 and Scintrex CG5-1184
- Additional observations between the above campaigns 2004-14, LaCoste & Romberg G54 and Scintrex CG5-740 and CG5-198

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- Observations from the NKG land uplift gravity lines 1975-2003. We chose to use only the observations from LaCoste & Romberg model G gravimeters
- Additional observations 1975-81, LaCoste & Romberg G54 and G290



**Figure 6:** *Left* Relative observations with the LaCoste & Romberg G54 in Karesuando AA. *Right* with the Scintrex CG5-1184 in Umbukta A, Sweden.

### 3.4 Vertical gravity gradient determination



**Figure 7:** *Gravity gradient measurements in Östersund AB, Sweden.*

When using the measurements at our absolute gravity points as a basis for relative gravimetry, the absolute gravity value normally refers to a bolt (or similar) on the ground. On the other hand, when measuring with and comparing different FG5 instruments, the gravity value is normally given 1.200 meters above the ground. For transferring the value from 1.200 meters to 0,000 meters as accurately as possible, one must measure the so called true vertical gravity gradient (Figure 7). In its easiest form, this means measuring the gravity difference between as close to 0.000 meters as possible and as close to 1.200 as possible with a relative gravimeter.

To get the lowest possible uncertainty, one has to measure with several different setups and at different heights and using a second-degree polynomial function. This is the way gradient measurements are performed at our Class A points, but for the Class B points a different approach was used (Engfeldt 2016a); Based on previous experience, the difference between e.g. three setups and one setup (close to the correct sensor height, which means around 70 cm for an A10) is very small and usually negligible. So, here the gravity gradient measurements were performed only with one setup measuring twelve independent differences.

### 3.5 Classification of the points

In total, 399 points were included in the two steps in the adjustment (see 5.2). Of these, 368 are classified according to the four classes in Table 1. The 31 not classified points are either destroyed or situated in Norway or Denmark.

*Table 1: The classification of RG 2000 points*

|         | Number of points | Description                                                                                                                      |
|---------|------------------|----------------------------------------------------------------------------------------------------------------------------------|
| Class A | 17               | Observed with the FG5                                                                                                            |
| Class B | 96               | Observed with the A10                                                                                                            |
| Class C | 181              | Observed with relative gravimeters, considered as very good in the main RG 2000 adjustment                                       |
| Class D | 74               | Observed with relative gravimeters, considered less suitable in the main RG 2000 adjustment or included in the second adjustment |

### 3.6 Comparisons and absolute gravimeter issues

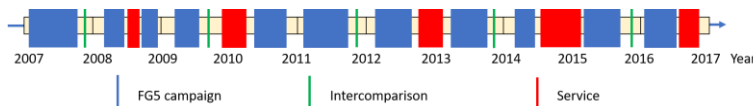
Just like all geodetic observations also absolute gravity observations are afflicted with errors, for instance errors related to the instrument, the software and/or the operator. These errors may occur as a random scatter or as a bias in the observed g-value and most of them tend to make the g-value lower (see e.g. Timmen et al 2014). It has been confirmed that something, at least once, happened with the FG5-233 during a service, introducing a new bias for the instrument (see below and Olsson et al 2015a). After a service, it is therefore very important to compare the instrument to another instrument whose difference to the FG5-233 was known before the service.

The FG5-233 has participated in several international absolute gravimeter comparisons (Figure 8). At all these, the weighted average of the participating instruments sets the level.

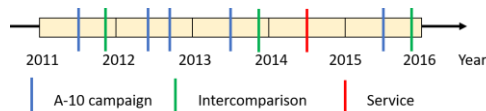
The FG5-233 has been five times on service at the manufacturer in USA, in the summer of 2008, after the field seasons in 2009, 2011, 2014, respectively, and after the summer 2016 when the instrument got upgraded to a FG5X system. After the service in 2009/10, a new bias was observed, with a shift larger than 4  $\mu$ Gal. This has been confirmed by all comparisons the

instrument has taken part in after that, but what caused it is still unknown. Whether something happened with the absolute level during the other three services is still under investigation, but if something happened the change was much smaller and much more difficult to determine with high significance. These issues are discussed in Olsson et al (2015a). After the upgrade to the FG5X, we suspect that a similar shift back about 4  $\mu\text{Gal}$  might have occurred, but this is not confirmed yet and will be evaluated through further absolute gravimeter comparisons.

To deal with these suspected offsets, different absolute levels for the instrument between the services could be assumed, which e.g. means that the level of the observations performed between autumn 2006 and summer 2008 should have an absolute level according to the ECAG2007, the level of the observations performed between autumn 2008 and winter 2009 should have an absolute level according to the ICAG2009 etc. This is further discussed in (Engfeldt 2016a). In the calculation and adjustment of RG 2000, all absolute gravity observations from the FG5-233 and the A10-020 were corrected according to the results of the ECAG and the ICAG offsets.



**Figure 8:** Overview of the FG5-233 observation periods, participation in intercomparisons and scheduled service.



**Figure 9:** Overview of the A10-020 observation periods in Sweden, participation in intercomparisons and scheduled service.

#### 4. DEFINITION AND REALIZATION OF RG 2000

RG 2000 are defined as follows:

- The gravity reference level as obtained by absolute gravity observations according to international standards and conventions
- The post glacial rebound epoch is 2000.0
- It is a zero permanent tide system

In the realization of RG 2000, each point realized by its gravity values and the standard uncertainties (obtained e.g. from the adjustment). This realization should not be viewed as closed. It will be possible to determine new sites in the future. Of course, this will require that we take care of the land uplift effect with sufficient accuracy, but as the models in question will improve with time, this is not expected to be a significant problem. Therefore, none of the points in RG 2000 (Figure 10) is regarded as perfect (free from errors). It will further be possible to

include new absolute gravity points in the future, that might be more accurate than the present ones.

The chosen and used land uplift model is NKG2016LU\_abs (Vestøl et al 2016). To convert the absolute land uplift to gravity change, the relation  $-0.163 \mu\text{Gal}/\text{mm}$  was used (Olsson et al 2015b). The A10-020 and FG5-233 final values were corrected with results from international comparisons (Olsson et al 2015a).

## 5. CALCULATION AND ADJUSTMENT OF RG 2000

### 5.1 Software

As the software used for RG 82 is not available anymore and a market check did not show a suitable software that fulfilled of our requirements, we decided to develop a new software in which we used the same observation equations and adjustment theory as in our old RG 82 software. The software contains three parts, Gprep, Gad and Gcross. Gprep prepares the data from different instruments and several input files to two input files for Gad in the land uplift epoch 2000.0. One of the input files to Gad includes all relative observations and the other includes precomputed differences, which in this case means observations where we only have the result (the “known” difference) and no relative observations. Gad is the main software and makes the least squares adjustment. The input files for Gad are the two previously mentioned plus a file with the absolute gravity data. Gcross is used to make cross validations for the absolute gravity observations. Cross validation means that the absolute observations of one certain point was excluded in one solution and the  $g$  value for the point of that solution was subtracted from the  $g$  value of the same point with the absolute observation included.

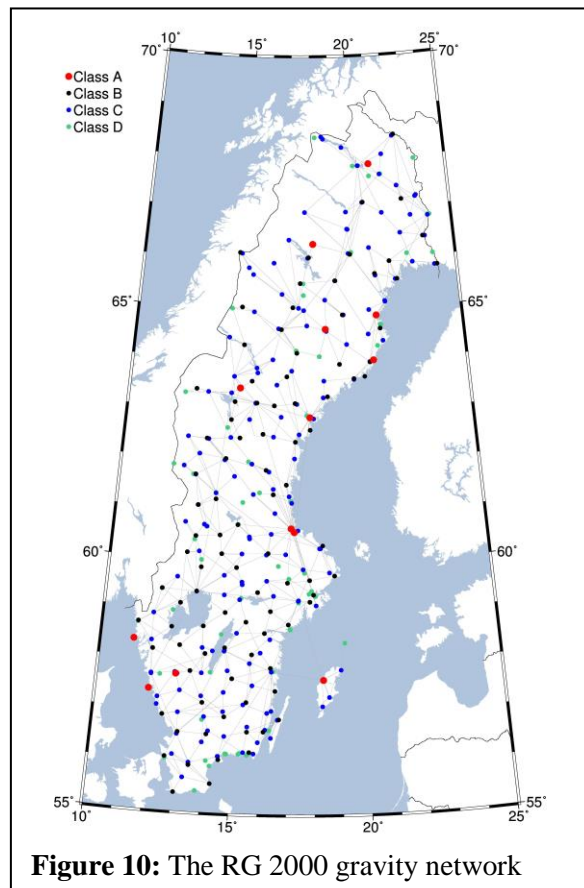


Figure 10: The RG 2000 gravity network

#### 5.1.1 Observation equations

The gravity values were adjusted by least squares adjustment. We had three different type of data and the formulas used can be seen below.

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1) Absolute gravity observations:

Absolute observation  $i$  of gravity point  $j$ , where  $l_i$  is the observed gravity value, observation  $i$ ;  $\varepsilon_i$  is the error of observation  $i$ ;  $g_j$  is the gravity value for point  $j$ :

$$l_i - \varepsilon_i = g_j$$

2) Relative gravity observations:

Relative observation  $i$  of gravity point  $j$  with instrument  $n$  for the instrument level  $k$  with drift parameter  $m$ , where  $l_i$  is the observed gravity value, observation  $i$ ;  $\varepsilon_i$  is the error of observation  $i$ ;  $g_j^0$  is the approximative gravity value for point  $j$ ;  $IL_k^0$  is the approximate instrument level for the sequence  $k$ ;  $t_i$  is the time of observation  $i$ ;  $t_k$  is the time for the first observation in sequence  $k$ ;  $SC_n^0$  is the approximate scale correction for the instrument  $n$ ;  $DR_m^0$  is the approximate drift parameter of the drift sequence  $m$ ;  $dg_j$  is the difference between the approximate and the "true" gravity value of gravity point  $j$ ;  $dIL_k$  is the difference between the approximate and "true" instrument level;  $dSC_n$  is the difference between the approximate and "true" scale correction:

$$l_i - \varepsilon_i = \frac{(g_j^0 - IL_k^0)}{SC_n^0} + (t_i - t_k) \cdot DR_m^0 + \frac{1}{SC_n^0} \cdot dg_j - \frac{1}{SC_n^0} \cdot dIL_k - \frac{(g_j^0 - IL_k^0)}{(SC_n^0)^2} \cdot dSC_n + (t_i - t_k) \cdot DR_m$$

3) Precomputed differences (was used for the published land uplift observations and where we only had old gravity differences, but no raw observations):

Observation  $i$  of difference between gravity point  $j$  and gravity point  $p$  estimating a scale correction for instrument  $n$ , where  $l_i$  is the observed gravity value, observation  $i$ ;  $\varepsilon_i$  is the error of observation  $i$ ;  $g_j^0$  is the approximative gravity value for point  $j$ ;  $g_p^0$  is the approximative gravity value for point  $p$ ;  $SC_n^0$  is the approximate scale correction for the instrument  $n$ ;  $dg_j$  is the difference between the approximate and the "true" gravity value of gravity point  $j$ ;  $dg_p$  is the difference between the approximate and the "true" gravity value of gravity point  $p$ ;  $dSC_n$  is the difference between the approximate and "true" scale correction:

$$l_i - \varepsilon_i = \frac{(g_j^0 - g_p^0)}{SC_n^0} + \frac{1}{SC_n^0} \cdot dg_j - \frac{1}{SC_n^0} \cdot dg_p - \frac{(g_j^0 - g_p^0)}{(SC_n^0)^2} \cdot dSC_n$$

## 5.2 Result from the main adjustment

In the main RG 2000 adjustment, the FG5 observations got the a priori standard uncertainty 1.0  $\mu\text{Gal}$ , the A10 observations got the a priori standard uncertainty 5.0  $\mu\text{Gal}$  and the relative observations got a priori standard uncertainties after certain criteria (normal value of 9.0  $\mu\text{Gal}$ ). Concerning the A10 observations and the relative observations, several different alternatives were tested before it was decided which a priori standard uncertainties should be used. 326

points were included in the main RG 2000 adjustment, of which 325 are in Sweden. The final solution (Figure 11) was run with an extra iteration which gave the identical result. During the adjustment, one gross error in one of the A10 observations was found and here the relative observations gave the point its g-value.

The second step adjustment was mainly done to get more connections between RG 2000 and RG 62, since about half of the observations are from 1973 connecting different RG 62 points of which some are Class B points in RG 2000. In this adjustment, the á priori standard uncertainties of the point previously determined in the main RG 2000 least squares adjustment were set to 0.1  $\mu$ Gal.

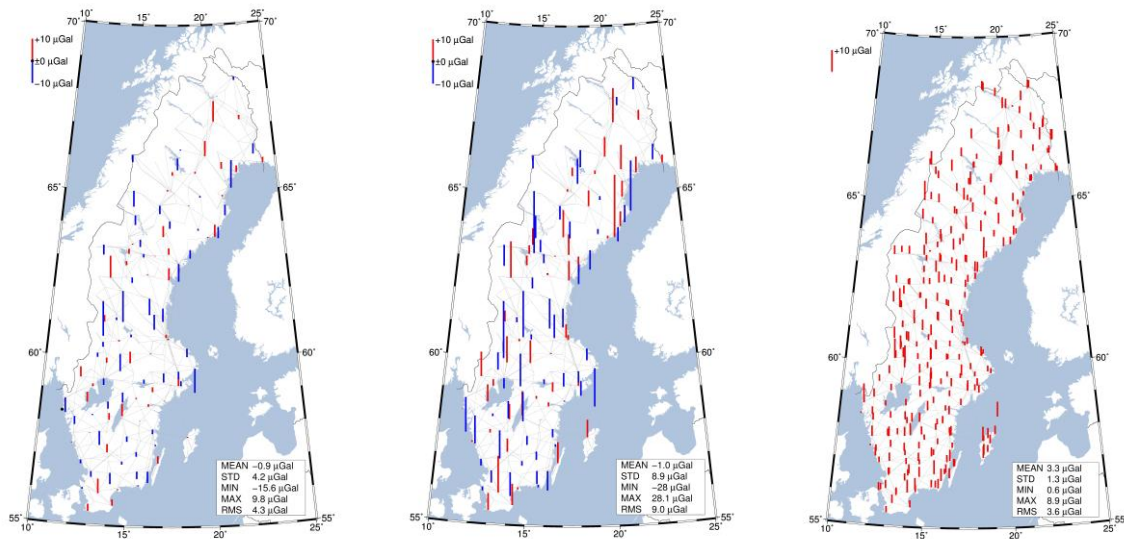
In the final solution of the main adjustment the total numbers were the following:

|                                         |      |
|-----------------------------------------|------|
| - Total number of observations:         | 1405 |
| - Total number of equations:            | 4008 |
| - Total number of absolute instruments: | 4    |
| - Total number of relative instruments: | 14   |
| - Number of gravity points:             | 329  |
| - Number of absolute observations:      | 113  |
| - Number of relative observations:      | 3721 |
| - Number of precomputed differences:    | 174  |
| - Number of unknown scale corrections:  | 9    |
| - Number of unknown drift parameters:   | 213  |
| - Number of unknown instrument levels:  | 854  |

The standard uncertainty of unit weight for

- All observations is 0.76
- The FG5 observations is 1.25
- The A10 observations is 1.32
- The relative observations is 0.74
- The precomputed differences is 0.68

It means that the weights were slightly overestimated for the absolute gravity observations and slightly underestimated for the relative observations. Still, we think that this is the best possible solution, as we for other reasons are convinced that the absolute observations are more trustworthy.



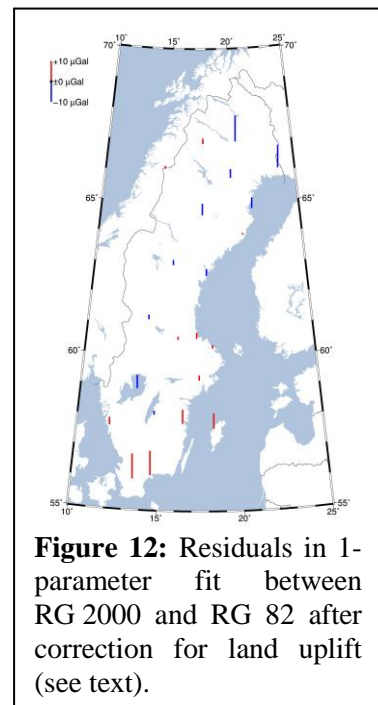
**Figure 11:** The results from the adjustment. Figure 11a shows the residual between observed AG value and adjusted value. Figure 11b shows the difference between residual of the result and the cross validation difference. Figure 11c shows the estimated uncertainty of all the 329 points in the main adjustment. The maximum value there is of a point which is destroyed.

### 5.3 Transformation between RG 2000 and RG 82

The work with transformations between RG 2000 and the previous reference frames are under progress. As a first we have derived a 1-parameter fit, after correction for land uplift, between RG 2000 and RG 82 based on 24 of the points included in the Zero Order Network of RG 82. The resolved transformation parameter is 28.2  $\mu$ Gal, and standard uncertainty in one common point 6.1  $\mu$ Gal. Residuals are shown in Figure 12.

## 6. SUMMARY/CONCLUSIONS

RG 2000 is the new gravity reference frame of Sweden and primary based on series of FG5 observations. The densification is done by A10 observations and relative observations. In the adjustment, none of the observations has been regarded as free from errors. The results show that the new RG 2000 do meet our expectations and will serve its purpose well. General conclusion is that future gravity reference frames should be based on



**Figure 12:** Residuals in 1-parameter fit between RG 2000 and RG 82 after correction for land uplift (see text).



absolute gravity measurements with absolute instruments periodically verified for their consistency with international gravity reference level.

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## BIOGRAPHICAL NOTES

MSc. Andreas Engfeldt is a geodesist at the Geodetic Infrastructure Department at Lantmäteriet, the Swedish mapping, cadastral and land registration authority. He has a MSc in Surveying and Mapping from the Royal Institute of Technology (Stockholm, Sweden) in 1997. He has been working at Lantmäteriet since 1997, with relative gravimetry since 1999 and with absolute gravimetry since 2004.

Dr. Martin Lidberg is head of the Reference frame section at the Geodetic Infrastructure Department at Lantmäteriet, the Swedish mapping, cadastral and land registration authority. He has a MSc in Surveying and mapping from the Royal Institute of Technology (Stockholm, Sweden) in 1988, and got his PhD from Chalmers University of Technology (Gothenburg, Sweden) in 2007. He has been working at Lantmäteriet since 1988. Martin is also a member of the EUREF Technical Working Group.

Dr. Per-Anders Olsson works with gravity surveys and postglacial land uplift research at the Geodetic Infrastructure Department at Lantmäteriet, the Swedish mapping, cadastral and land registration authority. He has a MSc in Surveying and mapping from the Royal Institute of Technology (Stockholm, Sweden) in 1998, and got his PhD from Chalmers University of Technology (Gothenburg, Sweden) in 2013.

Dr. Jonas Ågren is senior geodesist at the Geodetic Infrastructure Department at Lantmäteriet, the Swedish mapping, cadastral and land registration authority, and Docent in Geodesy at the Royal Institute of Technology (KTH) in Stockholm. He has a MSc in Surveying and Mapping from 1994 and got his PhD in Physical Geodesy in 2004, both at KTH. He has more than 20 years of experience of geoid calculation, gravimetry, land uplift modelling and height systems. He is the chair of the NKG Working Group of Geoid and Height Systems and leader of activity 2 in the FAMOS project.

Dr. Holger Steffen is a geophysicist at the Geodetic Infrastructure Department of Lantmäteriet, the Swedish mapping, cadastral and land registration authority. He holds a Diplom (MSc. equivalent) in Geophysics (2003) from the Friedrich Schiller University in Jena, Germany, and a PhD. (Dr. rer. nat., 2006) from the Free University of Berlin, Germany. After working for several years as a Postdoctoral researcher at universities in Germany, Canada, and Sweden, he joined Lantmäteriet in 2012. He is mainly working with glacial isostatic adjustment modelling.

Mr. Henrik Bryskhe works as a software engineer at the Geodetic Infrastructure Department at Lantmäteriet, the Swedish mapping, cadastral and land registration authority. He has worked in the field of medtech and studied engineering physics in Uppsala.

Prof. Jan Krynski, PhD in physics, DSc in technical sciences, is a full Professor and the head of the Centre of Geodesy and Geodynamics at the Institute of Geodesy and Cartography. His areas of expertise cover reference systems and frames in astronomy and geodesy, geodynamics,

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physical geodesy, gravity field modelling, space geodesy, global positioning systems. He is a member of multiple National and International Geodetic and Geophysical Associations and Committees and worked as a researcher and professor in multiple countries around the globe. He works at IGIK since 1998.

Dr. Marcin Sekowski is an assistant professor in the Centre of Geodesy and Geodynamics at the Institute of Geodesy and Cartography. He has a MSc in physics, specialization in astronomy from the University of Warsaw (1987), PhD in Geodesy at IGIK (2005). He specializes in reference systems and frames and the use of absolute gravimetry for the realization of reference frames. He works at IGIK since 1988.

MSc Przemyslaw Dykowski is the head of the Research-Calibration Gravimetric Laboratory of the Centre of Geodesy and Geodynamics at the Institute of Geodesy and Cartography. He has a MSc in geodesy from Warsaw University of Technology, specialization Geodesy and Satellite Navigation (2010). His areas of expertise include absolute and relative gravimetry and their application to the realization of gravity reference frames. He works at IGIK since 2011.

Dr. Emil Nielsen is a geophysicist at the Technical University of Denmark, DTU Space, in the Geodynamics division. He has a MSc in Geophysics from the GeoScience Institute at Aarhus University, and got a PhD from DTU Space in 2013. After his MSc he worked six months in the HydroGeophysics Group at Aarhus University. He has worked at DTU Space since 2008 with the main focus on gravimetry, geodynamics and geodesy.

## CONTACTS

MSc. Andreas Engfeldt  
Geodetic Infrastructure  
Lantmäteriet  
Gävle  
SWEDEN  
Tel. + 46 26 63 37 93  
Email: andreas.engfeldt@lm.se

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