Underground Azimuth Determinations Using an Adapted Wild GAK1

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Key words: Automated system, Gyro, Gyrotheodolite, Wild GAK1.

ABSTRACT

Mining and tunelling projects frequently involve the construction of long tunnels whose azimuths are to be determined very accurately, particularly prior to holing. Although conventional traverse methods may be employed, generally, these cannot guarantee the accuracy required and contractual conditions may then specify that independent gyrotheodolite bearings must be obtained. To this end, and as an alternative to the hiring of one of the modern automated systems, a theodolite with a Wild GAK 1 gyro attachment may be utilised. However, this instrument is difficult to operate with confidence, and good quality results are unlikely to be obtained without the involvement of surveying operators who are well experienced in the relevant technique.

This paper describes a fully automated system which is being developed at the Camborne School of Mines (CSM), University of Exeter: it includes an electronic 'eye' attachment to the Wild GAK1 gyro instrument, specifically designed to resolve the problems associated with manual observations and the recording of gyro tape oscillation data. The device incorporates two time-data capture sensors positioned at known points within the reflected swing path of the gyro tape's image, allowing sensor passing times to be recorded and automatically processed to establish the location of the tape's centre of oscillation; from which the true north azimuth of an unknown baseline can then be calculated.

A more complete account of the entire automated system is given comprising: a description of the sensors' structure and operation, the means by which the time data is captured and recorded, a brief account of the mathematical theory for the tape zero and spun up mode oscillation models and the new time data processing algorithm. A visual basic computer programme has been written to control each stage of the automatic procedure but details of this are not included here. Also, a summary of the calculation steps for the determination of the instrument's 'error factor' and a separate azimuth determination is given. The first of these is more specifically detailed in an actual example, with matters of accuracy being discussed as the calculation proceeds.

It is not claimed that the accuracy of results obtained by the use of this modified Wild GAK1 method are in any way superior to those obtained from modern Gyromat instruments but throughout this paper, the emphasis is on the description of an automatic system which is relatively simple to operate, accurate, reliable and comparatively inexpensive. Possible improvements in design, accuracy of calculation and adaptability are presently being investigated, with some of these being discussed in section 8.

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1. INTRODUCTION

The Wild GAK1 has its gyro suspended by a thin tape, with its spin axis horizontal due to the effect of gravity. The gyro rotor, spinning at 22,000 rpm, attempts to maintain the same initial, vertical, rotation plane created by its moment of inertia but, as it is situated on the surface of the earth, the influence of the earth's rotation prevents it from doing so. As a consequence of this constraint, the gyro's spin axis swings about the line of the tape (precession) and, at certain stages, it passes through the (true north) meridian plane where the gyro is then spinning as the earth, from west to east. Due to the gyro's angular momentum, its axis does not remain in the same meridian plane, but oscillates about it.

Conventional use of the gyro involves the observer viewing an image of the tape and either timing its passage through pre-selected markers within the viewing window, or estimating the location of the turning points of the tape's image, with respect to a graduated scale. Successful and accurate application of both of these methods depend upon the employment of careful and experienced operators, who are capable of obtaining and interpreting reliable data sets. A modification to the turning point estimation procedure has been developed by (Smith, 1980) with the inclusion of a micrometer, which permits a significant improvement in accuracy. However, although such methods frequently result in very accurate determinations, reliability is always questionable when the possibility of human error is involved. By use of the automated system incorporating the electronic 'eye' attachment (developed at CSM), neither manual timing nor distance estimation of oscillation characteristics is included, so the possibility of human error is mainly eliminated.

2. SYSTEM STRUCTURE

The system structure (Figure 1) has been designed to incorporate a relatively simple modification to the Wild GAK1 gyro so that, through the automatic acquisition of time data, the equilibrium position for the damped tape image motion (tape zero mode) or the centre of undamped oscillations (spun up mode) may be determined. Also, with the use of a detachable eyepiece, the conventional use of the gyro has not been compromised.

The standard Wild GAK1 power pack provides a 12V power supply to the modified system through the gyro and the signal conditioning and communication unit (SCCU) but, for the 5V krypton bulb which supplies a more responsive light to the sensors, a voltage regulator has been introduced between the power pack and the bulb to effect the required voltage reduction. A simple switch operation and bulb change returns this part of the system to its original function state. The sensor eyepiece device is attached to the instrument using the bayonet fixing that is normally used by the temporarily replaced observation tube, ensuring that the two photoelectric sensor holes inside the eye piece are parallel to the gyro oscillation scale.

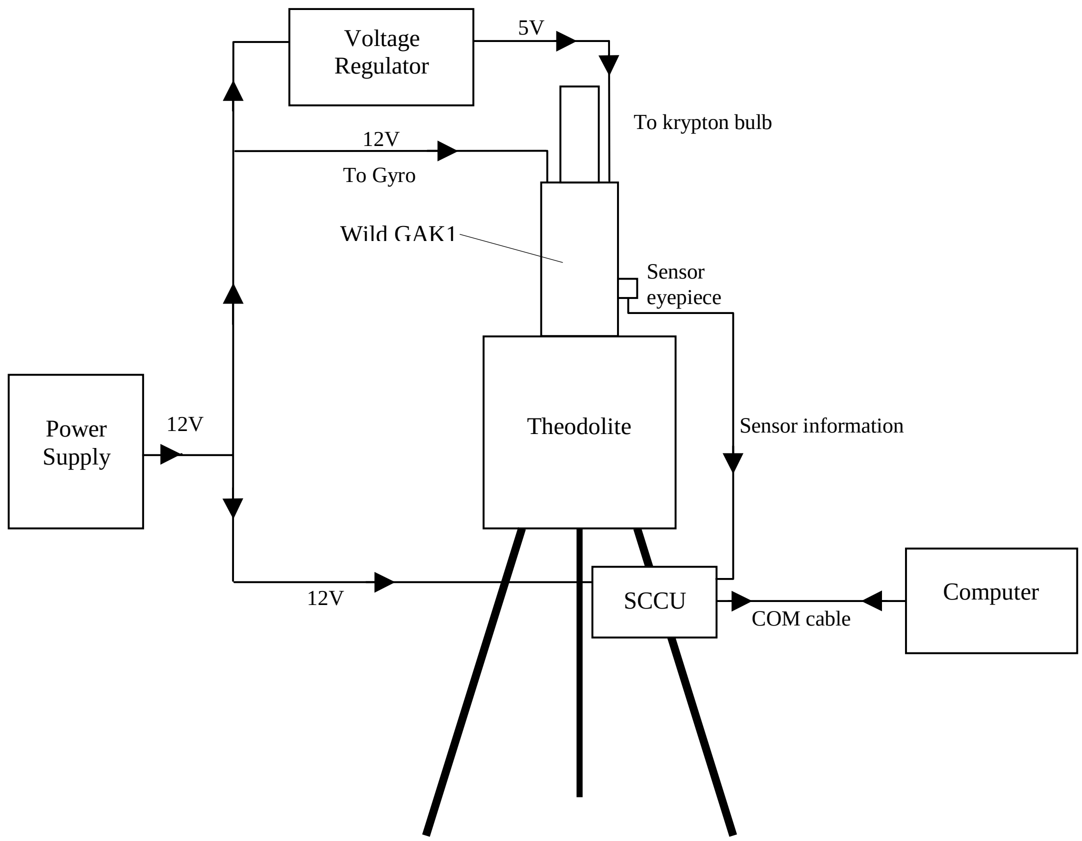

Figure 1 : Block diagram of system structure

The present system operates with two TSL 252 sensors, each having a 0.26mm^2 photosensitive area, and these are set behind two finely drilled circular holes, each of diameter 0.4mm, which are completely covered by the sensors. For convenience, in the following sections, the holes shall be referred to as the 'sensors'. The manner in which the sensors operate and provide information is described in the next section.

As the sensors respond to the movement of the gyro tape, signals are passed to the SCCU which then converts them to specific information about the position of the gyro tape, and this information can be sampled by means of a question and answer 'qa' trip code, set up between the SCCU and the computer through a two-way COM cable. This method of polling has been devised to avoid an excessive amount of data being sent from the SCCU to the computer and overflowing the COM port buffer.

For fieldwork, use of a ruggedised PC is preferred but, alternatively, a fairly robust laptop or palmtop computer may be used. The computer is programmed to convert the SCCU 'qa' data responses to internal clock time data, to process this data using an appropriate algorithm and to calculate, save and display any results or information that might be required immediately or at a later stage of the azimuthal determination. The computer language employed is Visual Basic.

3. OPERATION OF SENSORS AND SIGNAL CONVERSION

Figure 2(a) shows a view through the eyepiece of the placement of the sensors in relation to the graduated distance scale, and a typical position of the reflected tape image during one of the gyro oscillations. Although not essential, it is customary to place the sensors so that they are positioned one each side of the scale centre, however, throughout the relevant time period involved, it is necessary that the tape’s swing passes beyond the sensors on both sides.

When no part of the sensor is obscured by the tape image, maximum light is received by the sensor and this amount of light (proportional to the area exposed) is converted to an amplified voltage; however, as the tape starts to pass over the sensor, the lighted area reduces with a corresponding reduction in the voltage. Figure 2(b) shows a sequence of positions of the tape relative to the sensor and, in particular, attention is drawn to the presence of a narrow slit down the middle of the tape; the effect of this slit on the voltage is shown by the slight bulge at the mid-point of the theoretical distance-voltage graph in figure 2(c). A precise formula can be derived for the relationship between the voltage and the distance of the centre of the tape from the centre of the sensor, but this is quite complex and its inclusion in this paper would not contribute to the general understanding of the sensor’s response to the tape’s motion.

Although the symmetrical graph shown in figure 2(c) is useful, for the method employed in this study, the final calculation process depends on the acquisition of time (rather than distance) data, and a view of the corresponding voltage-time graph is more helpful. Figure 2(d) shows such an idealised graph, with an asymmetrical bias appearing as a result of the changing speed of the tape during its passage over the sensor. (This particular graph indicates that the speed of the tape is increasing during the passage). Also, the diagram illustrates the effect of ‘noise’ on the graph, highlighting the difficulty of establishing the precise times at which the tape starts or stops obscuring the sensor. To partially obviate this problem and the possibility of interference from the lower bulge anomaly, voltage trigger levels are introduced to divide up the relevant time region into three ‘states’, denoted by 1, 2 and 3; which correspond loosely to both sensors being clear, the left-hand sensor being not clear and the right-hand sensor being not clear, respectively. When the voltage reaches one of the trigger levels, a signal is sent from the sensor to the SCCU and the state number is changed from the previous value to the new one.

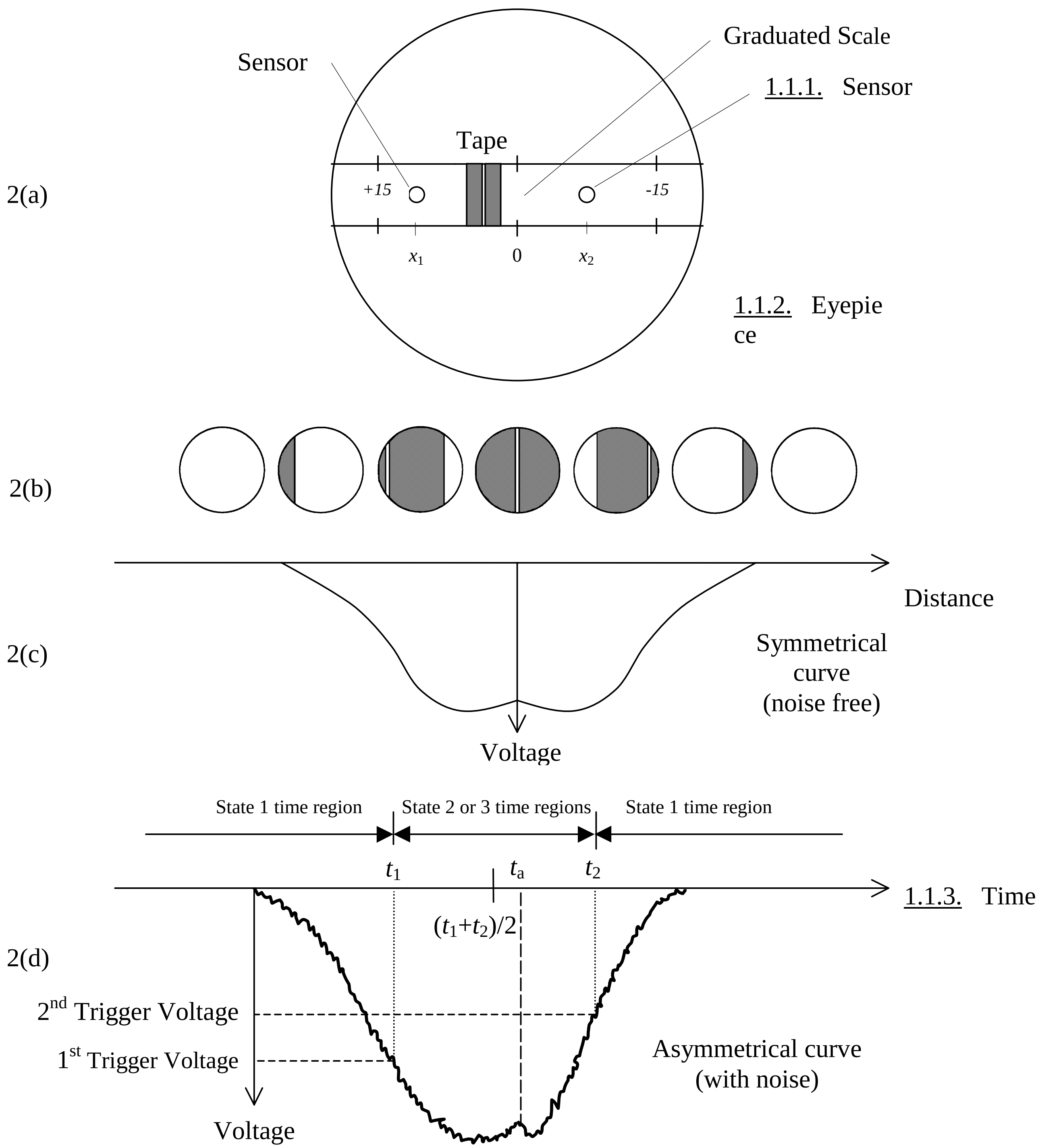

Figure 2 : Diagrams showing the relationship between the moving gyro tape and the sensors. Thus state 1 changing to state 2 indicates that the tape has 'started' to pass over the left-hand sensor, and state 2 changing to state 1 indicates that the tape has 'started' to leave the sensor; similarly for the right sensor with state 2 being replaced by state 3.

Via the SCCU, the computer's 'qa' system continuously samples the state of the sensors and, through a state memory protocol, which disregards any temporary noise-initiated state changes, the computer allocates times t_1 and t_2 to the first state changes from 1 to 2 and 2 to 1 (or 1 to 3 and 3 to 1) at the two assigned voltage trigger levels, respectively. The mean of these two times is then taken as the time when the centre of the tape coincides with the centre of the sensor. If the true time is t_a, then a particular source of error is apparent: figure 2(d) shows that, due to the variation in the tape's speed, the time t_a (at the peak of the lower bulge) differs from the $(t_1+t_2)/2$ position. It can be shown that, if the mathematical model for the tape's oscillatory motion is given by the equation

$$x = Ae^{-kt}\sin(\omega t + \alpha) \tag{1}$$

where A, k, ω and α are initially unknown parameters then, to the second order of smallness of (t_2-t_1),

$$t_a \cong \left(\frac{t_1 + t_2}{2}\right) - \frac{1}{8}\left[\frac{\left(k^2 - \omega^2\right)\tan\left(\omega t_1 + \alpha\right) - 2\omega k}{k\tan\left(\omega t_1 + \alpha\right) - \omega}\right]\left(t_2 - t_1\right)^2 + F \tag{2}$$

where F represents other independent time errors.

To investigate the effect of the speed variation error, tape zero equilibrium position determinations were derived from sets of mean time data and the corresponding time data sets adjusted by the term given in equation (2). It was found that the final results differed by less than 0.1%, indicating that this source of error is not significant.

Another source of error (included within F) relates to the use of the mean time $(t_1+t_2)/2$ though t_1 and t_2 correspond to different voltage trigger levels. This offset voltage device is included in the system design so as to introduce hysteresis into the circuit and thereby reduce the effect of noise, but experimental measurements have shown that, although voltage trigger levels may differ by up to about 5%, the corresponding t_2 time errors are less than 1%. As before, such errors create no significant change in the calculated value of the tape's equilibrium position.

Despite the existence of many additional sources of error throughout the entire data acquisition and processing procedure, it has been demonstrated (Wetherelt and Hunt, 2001) that, by the method described here, the final results are very accurate, reliable and quickly obtained. Nevertheless, research is continuing, to reduce errors from as many sources as possible and, generally, to introduce improvements into all aspects of the method and procedure; further details of some of these areas of research are given in section 8.

4. MATHEMATICAL THEORY

Equation (1) is the solution of the governing mathematical model formulated from the assumptions that the magnitudes of damped oscillations are relatively small and that the damping forces are negatively proportional to the velocity (Wetherelt and Hunt, 2001). Thus, in the case that the motion of the tape is initially right to left and the time is taken as zero when the tape centre first coincides with the centre of the left-hand sensor, then the following set of equations obtain:

$$x_1 = A\sin(\alpha) \tag{3}$$

$$x_1 = Ae^{-kt_1}\sin(\omega t_1 + \alpha)$$

$$x_2 = Ae^{-kt_2}\sin(\omega t_2 + \alpha)$$

$$x_2 = Ae^{-kt_3}\sin(\omega t_3 + \alpha)$$

$$x_1 = Ae^{-kt_4}\sin(\omega t_4 + \alpha)$$

$$s = x_1 - x_2\,,$$

where 0, t_1, t_2, t_3 and t_4 are the times when the tape's centre passes through the centres of consecutive sensors and s is the known scale distance between the centres of the sensors. At this stage in the calculation, x_1, x_2, A, k, ω and α are unknown.

Although equations (3) cannot be solved precisely for the six unknowns, it has been shown (Wetherelt and Hunt, 2001) that, for any set of times t_1, t_2, t_3 and t_4, an iterative method can be employed to do so, thus permitting the determination of the oscillation's equilibrium position to a high level of accuracy. For sets of times recorded from different sensor passing sequences, similar methods of solution are employed and, if N (≥5) is the number of sequential sensor passing times recorded, then N-4 determinations of the equilibrium position can be obtained, with the mean of these being taken as the final result to be used in further calculations. The standard deviation for these values provides a measure of the reliability of the final result. No precise criteria for convergence of the iterative process have been established but it seems certain that, providing the time data is sufficiently accurate to reflect the general characteristics of damped harmonic motion, then convergence follows. This has been so for all tape zero examples considered by the authors.

To determine the equilibrium position for spun up mode oscillations, one is tempted to assume that the same procedure as that for tape zero may be employed since some degree of damping, no matter how small, must be present in the system. Indeed, many excellent results have been obtained for the spun up mode by this method, however, in other cases, the iterative process has failed to converge. In the search for possible causes of such inconsistencies, it has been noted that for the spun up mode, the motion is very nearly simple harmonic and, if we set k equal to zero in equations (3), then we have more equations than unknowns. However, from these equations, we can deduce the linear relationship $t_1+t_4 = t_2+t_3$ between the times. It follows that, if this is so (or very nearly so), then the use of the iterative method is inappropriate as its convergence depends upon the inclusion of a damping component and six independent equations in the calculation procedure. Also, if the above time relationship is very nearly satisfied, then small variations in individual times due, for example, to noise or slightly inaccurate measurement, could result in the creation of a consecutive group of five times which do not correspond to the assumed damped harmonic motion model - as successful application of the iterative method requires.

In view of the lack of reliability of the iterative method when applied to spun up mode time data, the authors have decided that a consistently reliable, alternative calculation method should be employed; and this has been incorporated in the automatic system's controlling computer programme. Appropriate application of the following equations permits determination of the centre of oscillation for sets of five consecutive time data (zeroed at the first), for the spun up mode:

$$V_1 = s\cos\left(\frac{\pi t_1}{t_4}\right)/2\sin\left(\frac{\pi t_2}{t_4}\right)\sin\left(\frac{\pi(t_2 - t_1)}{t_4}\right) \tag{4}$$

$$V_2 = s\cos\left(\frac{\pi(t_1 + t_2)}{t_4}\right)/2\sin\left(\frac{\pi t_1}{t_4}\right)\sin\left(\frac{\pi t_2}{t_4}\right) \tag{5}$$

Taking the distances of the left-hand and right-hand sensors from the scale centre as S_L and S_R, respectively (giving regard to sign), then the distance D of the centre of oscillation from the scale centre is given by

$$D = S_L - V_1 \tag{6a}$$

when the five passing times correspond to the order of sensors LLRRL,

$$D = S_L + V_2 \tag{6b}$$

when the sensor order is LRRLL,

$$D = S_R + V_1 \tag{6c}$$

when the sensor order is RRLLR and

$$D = S_R - V_2 \tag{6d}$$

when the sensor order is RLLRR.

For the 6(a) and 6(c) cases, alternative (but very nearly equal) values of V_1 may be used by replacing t_4 by $t_2 + t_3 - t_1$ whereas, for cases 6(b) and 6(d) no such alternative calculation is possible for V_2. This variation is not included in the computer programme as differences in V_1 are not significant, even though for 6(a) and 6(c), only four consecutive sensor times are necessary to calculate D. This omission also permits simplification of the programme structure.

As described in the next section, to determine the azimuth of a specific line in a tunnel, using a gyro theodolite, two sets of tape zero and at least two sets of spun up mode time data are normally required. Details of the calculations' background theory are not given here, as they are familiar to most mining surveyors or are easily accessible for reference purposes (Thomas and Asquith, 1975) and (Thomas, 1976).

5. AZIMUTH DETERMINATION PROCEDURE

When using a GAK1 gyro theodolite to determine a very accurate value of the true north azimuth of an unknown line, two distinct operational techniques are usually involved: namely, the tape zero and the spun up procedures. Conventionally, tape zero data is obtained first, then several sets of spun up mode data and, finally, another tape zero set is obtained. As described in what follows, two important constants are involved in the subsequent calculations, viz. C, which relates the theodolite angle measurements to the corresponding scale measurements, and E, which is the horizontal angle between the meridian plane and the direction of the line of sight of the theodolite telescope in relation to to the centre line of the gyro oscillations.

The tape zero position is defined as the equilibrium position of the damped oscillation relative to the instrument, with the gyro *not spinning* and, in this theoretical position, the torque due to the suspension tape and to the power leads, acting on the oscillating system, would cancel out. For reasons of reliability it has been found, in practice, that the tape zero determination is best conducted along the same pointing as that of the approximate true north which, for the gyro theodolite, can be obtained from a compass reading or, more accurately, from the standard reversal method. To conduct a tape zero determination with the automatic system, the gyro clamp is gently lowered manually and the computer programme started. The periodic times for tape zero oscillations are approximately 60 seconds.

For the spun up mode procedure, it is necessary to align the gyro theodolite so that it is close to true north, i.e. less than one degree west or east of it; and the corresponding gyro theodolite angle reading of this selected pointing must be recorded. To start, the gyro is *spun up* to its operational speed and, when this is reached, the gyro clamp is carefully lowered and the computer programme started. The periodic time for a spun up mode oscillation is approximately 430 seconds. The purpose of conducting a spun up mode procedure is to calculate the position of the centre of oscillation (for an assumed undamped simple harmonic motion) so that, from the results obtained from two such determinations, the constant C may be derived although, in practice, several such C determinations are made and these are employed in later error factor calculations.

A summary of the procedure and calculation steps to determine the azimuth of an unknown line are as follows:

(i) The line's backsight theodolite reading (V) is recorded. This can be checked at later stages of the data acquisition procedure to ensure that no slipping of angle has occurred.
(ii) Initial tape zero determinations are made at the approximate true north position and the mean scale value (d_1) of these is recorded.
(iii) Spun up mode determinations are made at two pointings on either side of the estimated true north direction, with an angular separation of about 30′ to 90′. The pairs of readings of

corresponding theodolite and centre of oscillation scale readings (ϕ_1, ϕ_2 and D_1, D_2) are recorded. The dimensionless constant (C) is now calculated from the equation $1+C = (\phi_2-\phi_1)/p(D_1-D_2)$, where the constant p is the value of one scale division in angular measure which converts the scale readings (D_1, D_2) into angular units; for example, if the ϕ readings are in degrees, then p = 1/6 and, if the readings are in minutes, then p = 10, etc. By taking suitable pair combinations from several such pointings, it is possible to derive a number of C value determinations from which, the same number of E evaluations may then be made, as described in sections (v) and (vi) below. It has been shown (Thomas, 1976) that C varies inversly as the cosine of the latitude, consequetly, local site variations have no significant effect on this parameter.

(iv) A second tape zero determination is made, as before, giving mean scale value d_2 and an appropriately weighted mean value (d) of d_1 and d_2 is recorded.

(v) For a typical pointing with parameters ϕ and mean D, the gyrotheodolite reading (W) of true north can now be calculated from the equation $W = \phi+p(D(1+C)-Cd)$ and it follows that the gyro bearing of the backsight is given by V-W.

(vi) If the ‘error factor’ (E) is known, then the true bearing (R) of the backsight is given by R = V-W-E, thus establishing a new baseline.

(vii) Step (i) is repeated to check that the initial backsight theodolite reading has not changed.

If the error factor is not known, then the entire procedure (with the variations employed in the example below) must be carried out, using a known baseline. The main difference between the two procedures is that if R is known from the outset and E is not, then E is determined from E = V-W-R, which is a transposed form of the equation given in (vi).

6. EXAMPLE

CSM Test Mine (part of the Camborne School of Mines), was chosen for this exercise and a single second Wild T1000 theodolite, with a GAK1 gyro and electronic eyepiece attachments, was deployed. The objective was to establish the theodolite’s error factor E from a known baseline (on the surface), so that one could then proceed to the determination of the true north bearing of an unknown, below surface, survey line**.** In this example, the true north bearing of the known baseline was recorded as 317° 06′ 53″ (= R); and the calculation stages carried out below follow the order described in the previous section:

(i) The baseline’s backsight theodolite reading was recorded as 18° 01′ 02″ (= V).

(ii) From V and R, true north’s theodolite reading was found to be 60° 54′ 09″, and this was chosen as the direction for the first tape zero determination. [It is important to note that, in general, the true north’s theodolite reading does not correspond to the zero mark on the gyro’s scale.] From 12 consecutive sensor passing times, the automatic system calculated 8 d-values, with mean 1.355105 (= d_1). The standard deviation of the d’s was 0.00812, indicating considerable accuracy and reliability.

(iii) Five pointings (denoted by P, Q, R, S and T) were selected around true north, and spun up mode determinations were made in each of these directions. The automatically calculated D values and other information is presented in the following table:

Table 1

Pointing	Theo. reading ϕ	Number of time data and D determinations	Mean D	Standard deviation
P	60º 10′ 14″	12 (8)	3.3070936	0.0114
Q	60º 30′ 50″	12 (8)	1.7181321	0.0134
R	61º 10′ 39″	13 (9)	-1.3471725	0.0097
S	61º 20′ 12″	12 (8)	-2.1074867	0.0216
T	61º 30′ 46″	12 (8)	-2.9075680	0.0089

Six pairs of pointings P-R, P-S, P-T, Q-R, Q-S and Q-T were taken, and their corresponding 1+C values were calculated. These are shown in table 2 below. It is observed that the mean of the 1+C values is 1.29519 and their standard deviation is 0.003024, again indicating that very consistent results have been obtained.

(iv) A second tape zero determination was made at the true north pointing, and 12 consecutive sensor passing times produced a mean d value of 1.424229 (= d_2), with a standard deviation of 0.01040, such differences in d_1 and d_2 occur frequently in practice, and it is customary to take a weighted mean of the two values to obtain a working value of d. In this case, a 1 to 2 weighting produced a mean of 1.401188 (= d).

(v) From each of the 6 pairs of pointings in (iii), the corresponding 1+C value and the two ϕ and D values, for any one of the pair of pointings, are used in the calculation of the 6 corresponding values of W. These are also shown in table 2. The mean of the W values is 60° 48′ 56.51″ and their standard deviation is 2.08″.

(vi) The working value of E can now be obtained from the relationship: Mean E = V-R-Mean W +360° (and the standard deviation of E is the same as that of W) but, for the sake of interest, the individual E values are also given in table 2. Thus, the final working error factor is taken to be 5′ 12.49″

Table 2

Pointing	1+C	W	Pointing	ϕ	D	E

pair		(+60° 48′ 50″)	choice			
P-R	1.29809	9.14″	P	60° 10′ 14″	As	5′ 09.86″
P-S	1.29219	2.39″	P	,,	shown	5′ 16.61″
P-T	1.29586	6.59″	P	,,	in	5′ 12.41″
Q-R	1.29895	7.73″	Q	60° 30′ 50″	table	5′ 11.27″
Q-S	1.29042	6.11″	Q	,,	1	5′ 12.89″
Q-T	1.29560	7.09″	Q	,,		5′ 11.91″

Mean E = 5′ 12.49″

(vii) A final check on the baseline's theodolite reading was made and no change was detected.

To continue the exercise, for the determination of the true north bearing of an underground survey line, similar steps are now repeated but with slight variations, as explained in the previous section.

7. CONCLUSIONS

This paper has outlined a new approach to underground azimuth determinations, using a simple electronic 'eye' attachment to a standard gyrotheodolite combination. In the early stages of research, the main objective was to develop a relatively inexpensive, fully automated system which captures and processes field data with a minimum of external intervention, producing results which are more accurate and reliable than those obtained through the application of conventional methods. The authors believe that the evidence presented here strongly supports the claim that considerable progress towards that objective has been made.

However, problems associated with some aspects of the development programme are still unresolved and work is continuing, to ensure that these difficulties are successfully overcome in the near future, (see section 8). It is anticipated that, ultimately, a comprehensive hardware and software package will be produced to control all stages of the procedure and, that this will make an important contribution towards the simplification of underground surveying projects.

8. CURRENT AREAS OF RESEARCH AND DEVELOPMENT

(a) Despite the accuracy of results presently being obtained, it is considered that use of simple TSL 252 sensors to detect the passage of the moving tape's image across the drilled holes, fails to exploit more than a very limited amount of the potential information available. For this reason, alternative use of very accurate linear sensor arrays is being investigated and, in particular, the TSL 1301 type array. This array consists of 102 sensors covering a total width of 8.6mm, which would readily fit into the 12mm of viewing space available.

Theoretically, at any time during the passage of the tape's image over the array, the voltage induced by the light exposure on each of the 102 sensors can be recorded, thus indicating the position of the tape relative to the array. For several (or many) such time-sampled sets of data, more than sufficient information should then be available to permit very accurate determination of location, velocity and acceleration of the tape's central section. Although it is possible to store all the available data before processing, in practice, a less data-compacting, sensor selection and time-sampling protocol will be employed.

Bearing in mind that the ultimate objective is to locate the position of the oscillation's equilibrium position, it might be possible (thought not yet established) to determine this position more directly by applying some appropriate numerical technique to the raw data, without making those original assumptions which led to the derivation of the fundamental equation of motion viz. Equation (1). In particular, the unjustified condition that resistance is negatively proportional to velocity could be relaxed, and other numerical calculation steps, required through the use of single sensors, would be unnecessary.

(b) Another important area of concern relates to the changes in the 'constant' d obtained before and after the spun up mode determinations, as described in stages (ii) and (iv) of section 5. For the example considered in section 6, the two d-values were found to be 1.355105 and 1.424229, respectively, which differ by about 5%. The causes of this inconstancy have been considered by a number of investigators, including (Smith 1980) but, as far as the authors are aware, no precise set of factors have been established; consequently, no completely reliable practical procedures or arithmetical adjustment factors have been recommended to overcome this problem.

A weighted mean of the 'before' and 'after' d-values is usually taken for future calculations, but the extent of weighting depends on operator 'experience', which may not constitute a very reliable accuracy criterion. As for the overall effect of this source of error on the final calculation of the error factor E: in our example, if either d_1 or d_2 had been taken as d then the maximum possible theoretical error in E would have been $pC(d_2 - d_1)$, which is equal to $600 \times 1.2952 \times 0.069124 \cong 54''$; for the two to one weighting employed in our example, the maximum possible error is approximately 36" (two thirds of the previous result). To minimise the maximum possible theoretical error, the simple unweighted mean of d_1 and d_2 should be taken which, in our example, would have given a maximum possible error of approximately 27".

Since any one of the above possible maximum errors is significant, and therefore a matter of concern, the following rather radical approach which dispenses with the tape zero part of the procedure altogether, is suggested as a possible means of avoiding d errors - and indeed, any other tape zero finding errors:

8.1.1. From the equation $R = V - E - [\phi + p(D(1 + C) - Cd)] = [V - pD(1 + C)] - [E - pCd]$ so that

8.1.2.
8.1.3. $R = [V - pD(1 + C)] - E'$
(7)

where the new 'error factor' E' is the difference between the old error factor E and the unknown tape zero error term pCd. Thus, working with a known baseline (so that R is known), V, D and C can be determined and E' can then be calculated from equation (7). If the gyrotheodolite is now set up on another line (possibly underground) of unknown azimuth and we assume that E and the pCd terms retain their previous values, for the new V,D and previously calculated E', equation (7) gives R, the true bearing of the unknown baseline.

Some experimental work has already been carried out, but a thorough programme of investigation to assess the accuracy and reliability of the above procedure is still necessary; theroetically it is a feasible method which is worthy of further investigation.

(c) When there is a manual change-over from spun-up mode to tape-zero mode (or vice versa), there is considerable risk of interference with the stability of the tape's oscillatory motion. Operators are advised to 'gently' lower (or raise) the gyro clamp but, generally, there is sufficient vibrational disturbance introduced into the system to create slippage or even movement of the theodolite's levelling system. It has been difficult to assess the extent to which such manual operations affect the stability and hence, the parameters of the system, but the authors are of the view that the introduction of a servo-assisted clamp lowering/raising attachment would eliminate one of the existing sources of error. The operation and design of such an attachment is presently being considered.

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