A Local Quasigeoid Determination Approach for Jawa Island (Indonesia)

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Key words:

SUMMARY

In the case of accurate local geoid determination for island of Jawa, there are some peculiarities which have to be taken into account when gravity anomaly data are combined with a global geopotential model. The gravity data sets contain systematic errors due to different height and gravity datums, and the availability of a detailed crust's density information of the island is very limited. On the other hand, the new gravity field missions CHAMP, GRACE and GOCE will provide a very accurate global model of the earth's gravity field. An alternative way to improve the situation is to apply a suitable choice of modified Stokes kernel in the combination solution of global geopotential model and gravity data based on Molodensky's approach. In this case, we get quasigeoid instead of the geoid. A promising potential candidate in modifying Stokes kernel is by means of the application of Butterworth filter.

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1. INTRODUCTION

Jawa is the most densely populated island in the Indonesian archipelago where more than half of the country's population inhabit the island. As a consequence, the living environment quality due to the over population within the island is degrading. In this case, accurate spatial information related to regional and city planning is become a necessity. One of them is topography information. By using modern geodetic space techniques such as GPS and INSAR, accurate orthometric height of the topography (digital elevation model or DEM) can be determined very fast and inexpensive. However, an accurate detailed geoid within the island is still not available yet.

A local geoid is usually determined gravimetrically. It is calculated based on gravity anomaly data. In view of Stokes approach, the gravity anomaly data are defined on the geoid surface [Heiskanen & Moritz, 1967]. Prior to geoid computation, the gravity data, observed on the earth's surface or at a certain flying height, have to be reduced to geoid surface. In this case, a good knowledge of crust's density is required. As we know that the mountainous Jawa Island is located close to subduction zone thus a complex mass density structure within the island could be encountered. However, the detailed density information availability in this island is still very limited. As an alternative to the Stokes approach, the use of Molodenski's boundary value problem solution could be more suitable [Molodenski et al, 1962]. The main advantage of this approach is the independency of the mass density knowledge, i.e. gravity reduction is not necessary. In this way we get the so called quasigeoid instead of the geoid. The distance from reference ellipsoid to the quasigeoid is known as height anomaly, whereas geoid undulation is known as a distance from reference ellipsoid to the geoid. Of course it is possible to transform the height anomaly to the geoid undulation but again the knowledge of mass density is required.

Other constraints in precise geoid determination in the island of Jawa are data availability and vertical datum unification problems [Prijatna, 1998]. Recently the gravity data covering the archipelago and its surroundings can be found in GETECH's database. All available land and marine gravity, and satellite altimetry derived gravity data were compiled, processed and stored in a unified data set. Due to the data availability and quality, this data set cannot be used for precise geoid determination purpose. It was derived mainly for other geophysical applications such as regional geological interpretations, basin analyses, and continental margin studies [GETECH, 1997]. Therefore, appropriate new gravity measurements covering the whole island of Jawa is strongly recommended. Another problem is lack of unified vertical height datum in the archipelago. The inconsistencies in height datum which are inherent in topography data will introduce long-wavelength errors on the estimated geoid heights [Heck, 1980]. An alternative way to reduce such effects is by means of a suitable choice of modified Stokes' kernel in combination solution of spherical harmonic potential coefficients and gravity data [Vanicek & Featherstone, 1998]. The modified kernel should also preserve as much as possible the low-frequency information contained in the global geopotential model. This is important since in the coming future the new gravity field missions such as CHAMP, GRACE and GOCE will provide a very accurate global geoid model to 1-2 cm at a spatial resolution of about 100 km [ESA (1999) and Rummel et al, 2002].

This paper is focused on a proposed computation approach suitable for the Jawa Island's geoid determination based on combination of global geopotential model and gravity anomaly data using Molodenski's approach.

2. MOLODENSKI'S FORMULA

The height anomaly ζ at point $P(\varphi,\lambda,r)$ defined in geocentric coordinate system can be computed using following formula [Molodenski et al, 1962 and Heiskanen & Moritz, 1967],

$$\zeta(\varphi,\lambda,r)=\frac{R}{4\pi\gamma}\iint_{\sigma}(\Delta g+G_1)S(\psi)\,d\sigma \qquad (1)$$

where R is the earth's radius, γ is normal gravity, ψ is the spherical distance from the computation point to the integration point, $S(\psi)$ is Stokes' function, Δg is free-air gravity anomaly which refers to ground level, and the simplified G_1 term may be written as,

$$G_1=\frac{R^2}{2\pi}\iint_{\sigma}\frac{h-h_p}{l_0^3}\Delta g\,d\sigma \qquad (2)$$

where l_0 is spherical distance, and $(h-h_p)$ is height difference between data point and computation point.

To transform the height anomaly ζ to geoid undulation N can be done by the following relation,

$$N\approx\zeta+\frac{\Delta g_B\,H}{\bar{\gamma}} \qquad (3)$$

where Δg_B is the Bouguer anomaly, H is height above sea level, and $\bar{\gamma}$ is the mean normal gravity.

3. COMBINATION OF GLOBAL GEPOTENTIAL MODEL AND GRAVITY ANOMALY DATA

Geoid undulation formula based on combination of global geopotential model and gravity anomaly data is derived. This application follows the recently proposed idea by Haagmans et al (2002) for a kind of multi-resolution concept for validation of GOCE gradiometry results based on regional gravity data. In our case, we combine a low-pass filtered global geopotential model contribution and a high-pass regional gravity anomaly contribution. The complete height anomaly can be expressed in an infinite series of spherical harmonics, and divide it into two separate parts, i.e. long- and short-wavelength parts, ζ^L and ζ^S respectively, as

$$\zeta(\varphi,\lambda,r) = \zeta^L + \zeta^S \tag{4}$$

$$\zeta^L = \frac{1}{\gamma}\sum_{n=2}^{\infty}\left(\frac{R}{r}\right)^{n+1} w_n \, T_n(\varphi,\lambda,R) \tag{5}$$

$$\zeta^S = \frac{1}{\gamma}\sum_{n=2}^{\infty}\left(\frac{R}{r}\right)^{n+1} (1-w_n) T_n(\varphi,\lambda,R) \tag{6}$$

where $T_n(\varphi,\lambda,R)$ is surface Laplace harmonics of the disturbing potential. In this case, the spectral weights or filters w_n fulfill $w_n + (1-w_n) = 1 \ \forall n$. This assures the full signal to be represented up to infinity.

The surface Laplace harmonics $T_n(\varphi,\lambda,R)$ is

$$T_n(\varphi,\lambda,R) = \frac{GM}{R}\sum_{m=-n}^{n}\overline{Y}_{nm}(\varphi,\lambda,R) \tag{7}$$

$$\overline{Y}_{nm} = \overline{C}_{nm}\,\overline{P}_{nm}(sin\,\varphi)\begin{cases} cos\,m\lambda & n \le 0 \\ sin\,m\lambda & n > 0 \end{cases} \tag{8}$$

where G is gravitational constant, M is mass of the earth, R is mean radius of the earth, $\overline{C}_{nm}$ are fully normalised disturbing coefficients, (n,m) are spherical harmonic degree and order, and $\overline{P}_{nm}(sin\,\varphi)$ are fully normalised Legendre functions.

The equation (4) can be regarded as a weighted combination of two complementary full global representations. The ζ^L is obtained from global geopotential model. While the ζ^S can be the detail refinements of the solution by gravity anomaly data. The relationship between ζ^S and Δg can be established straightforwardly by using the relation as follow,

$$\zeta^S = \frac{R}{\gamma}\sum_{n=2}^{\infty}\left(\frac{R}{r}\right)^{n+1}(1-w_n)\frac{1}{n-1}\Delta g_n(\varphi,\lambda,R) \tag{9}$$

where the $\Delta g_n(\varphi,\lambda,R)$ is surface Laplace harmonics of the gravity anomaly. By using orthogonality relation between Legendre polynomials over the sphere, an integral form of equation (9) which relates ζ^S and the observed gravity anomaly can be derived as,

$$\zeta^S = \frac{R}{4\pi\gamma}\int_{\sigma}\Delta g\,\bar{S}(\psi)\,d\sigma \tag{10}$$

The kernel function or the modified Stokes' function $\bar{S}(\psi)$ is,

$$\bar{S}(\psi) = \sum_{n=2}^{\infty}\frac{2n+1}{n-1}(1-w_n)P_n(cos\,\psi) \tag{11}$$

where $P_n(cos\,\psi)$ are Legendre polynomials.

In practice, we only evaluate the integral in the equation (10) numerically within a certain integration cap size σ_0 due to data coverage limitation, and neglect the part beyond the cap. Furthermore the summation in the equation (5) is also limited only up to a certain maximum degree n_{max}. As a consequence, two kinds of errors on the estimated height anomaly are introduced. First, due to the errorneous in both geopotential coefficients and gravity anomaly data, the total geoid commission error is yeilded. Secondly, the total omission error is also resulted. This is due to neglection of both gravity anomaly data outside integration cap and geopotential coefficients of $n > n_{max}$. Following the idea of derivation by de Min (1996) and Vanicek & Featherstone (1998), the total omission error can be approximated as,

$$\delta\zeta = \frac{1}{\gamma}\sum_{n=n_{max+1}}^{\infty}\left(\frac{R}{r}\right)^{n+1}\left[w_n + \frac{n-1}{2}Z_n(\psi_0)\right]T_n(\varphi,\lambda,R) \tag{12}$$

The terms $Z_n(\psi_0)$ are called as truncation coefficients,

$$Z_n(\psi_0) = \sum_{k=2}^{\infty}\frac{2k+1}{k-1}(1-w_k)E_{nk}(\psi_0) \tag{13}$$

where,

$$E_{nk}(\psi_0) = \int_{\psi=\psi_0}^{\pi} P_n(\cos\psi) P_k(\cos\psi) \sin\psi \, d\psi \qquad (14)$$

and the term,

$$w_n^T = w_n + \frac{n-1}{2} Z_n(\psi_0) \qquad (15)$$

The $E_{nk}(\psi_0)$ can be evaluated numerically by a recursive formula as described by Paul (1973). From equation (12) it is obvious that the magnitude of total omission error is directly controlled by the maximum degree of geopotential model used, the choice of spectral weights, and the integration cap size. For the reason of simplicity, discussion related to the detailed commission error is not yet included.

Assuming that all of the potential coefficients are uncorrelated, and based on a signal degree-variance model for gravity anomalies c_n, e.g. Tscherning-Rapp's model [see Tscherning and Rapp, 1978], the total omission error variance of height anomaly σ_ζ^2 can be estimated from,

$$\sigma_\zeta^2 = \frac{R^2}{4\gamma^2} \sum_{n=M+1}^{\infty} \left(\frac{R}{r}\right)^{n+3} \left[\frac{2}{n-1} w_n + Z_n(\psi_0)\right]^2 c_n \qquad (16)$$

where M is the maximum degree of the geopotential model used. Hence, the height anomaly $\bar{\bar{\zeta}}$ can be estimated from the following equation,

$$\bar{\bar{\zeta}} = \frac{1}{\gamma} \sum_{n=2}^{n_{max}} \left(\frac{R}{r}\right)^{n+1} \left[w_n + \frac{n-1}{2} Z_n(\psi_0)\right] T_n(\varphi, \lambda, R) + \frac{R}{4\pi\gamma} \iint_{\sigma_0} \Delta g \, \bar{S}(\psi) \, d\sigma \qquad (17)$$

where σ_0 indicates the cap size.

4. CHOICE OF SPECTRAL WEIGHTS AND INTEGRATION CAP SIZE

In order to fulfill the previous mentioned problem statement, we need a spectral weight model which has the following properties:

- it can be tuned to select which degrees are mainly used from global geopotential model,
- it stays close to one for lower degree n,
- it stays close to zero for higher degree n, and
- the kernel is exactly zero at and outside cap boundary.

In this case, two spectral weight model are compared, i.e. Wong-Gore [see Wong & Gore, 1969] and Butterworth filter models. Table 1 shows the formulation of both spectral weight w_n.

($n_{max} = 180$) and Butterworth filter ($n_b = 180$, $k = 12$) and by using equation (11), the corresponding kernel functions are shown in Figure 2. In this examples, we intend to preserve the long-wavelength signals from geopotential model up to about $n = 180$. On the other hand, the long-wavelength information contained in gravity anomaly data have to be removed while the short-wavelength parts are maintained. Applying equation (16) by means of the use of those two filters, we can compute the total height anomaly omission error variance for specific cap sizes. If we define $M = 360$, the results is shown in Figure 3. Choosing an omission error e.g. 5 cm would correspond in this example to cap sizes of approximately $\psi_0 = 4.24°$ (Wong-Gore's model) and $\psi_0 = 4.11°$ (Butterworth filter). These values correspond to one of the kernel function's zero-crossings, and it yields minimum value of the total omission error [de Witte, 1967 and Heck & Gruninger, 1987]. Figure 4 shows the total spectral weights of the two filters based on the chosen cap sizes.

Based on Figure 3 and Figure 4, it is clearly seen that the use of Butterworth filter is able to preserve the long-wavelength information from global geopotential model better, and it also yields smaller omission error. In addition, another important advantage to apply such filter type, the effect of long-wavelength error inherent in gravity anomaly data due to the vertical and gravity datums inconsistencies on the estimated height anomaly could be reduced more effective.

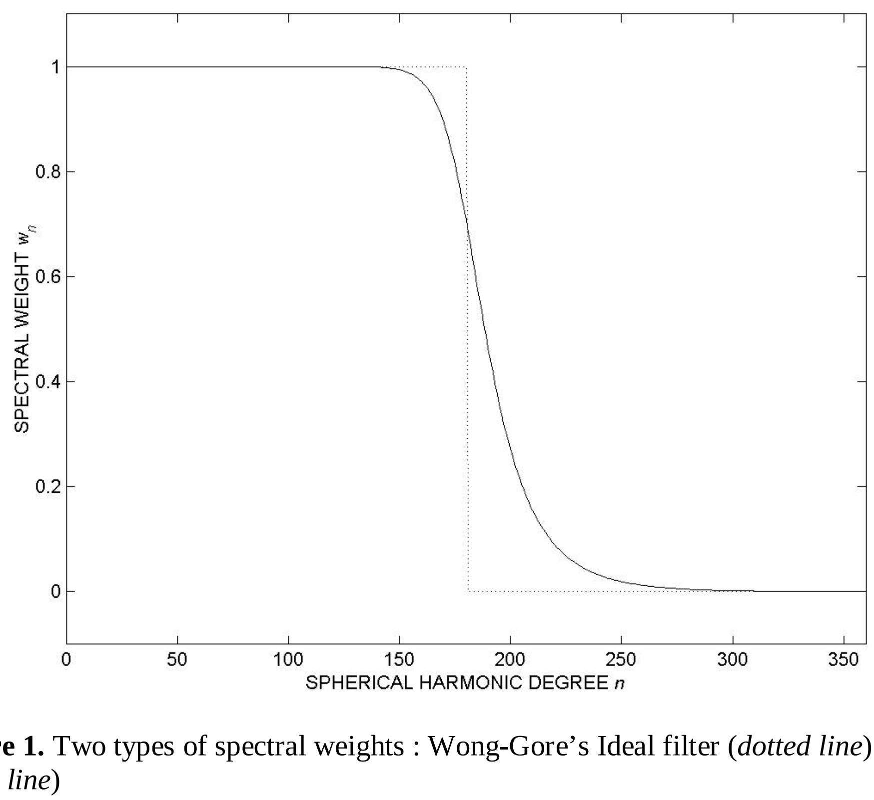

Figure 1. Two types of spectral weights : Wong-Gore's Ideal filter (*dotted line*) and Butterworth filter (*solid line*)

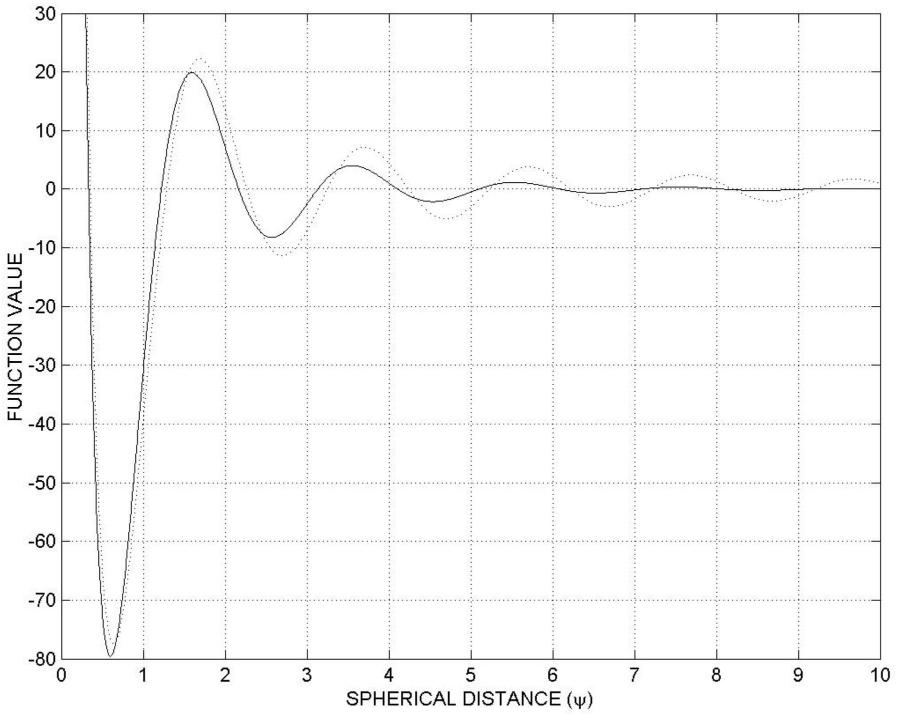

Figure 2. The corresponding kernel functions : Wong-Gore's Ideal filter (*dotted line*) and Butterworth filter (*solid line*)

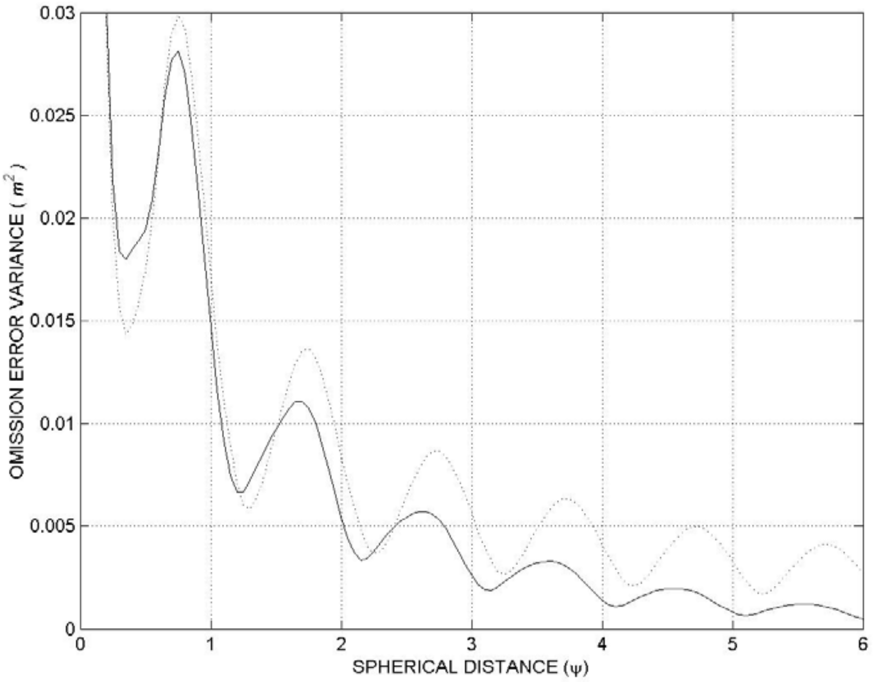

Figure 3. Total height anomaly omission error variance (m^2) for various cap sizes (degree) : Wong-Gore's Ideal filter (*dotted line*) and Butterworth filter (*solid line*)

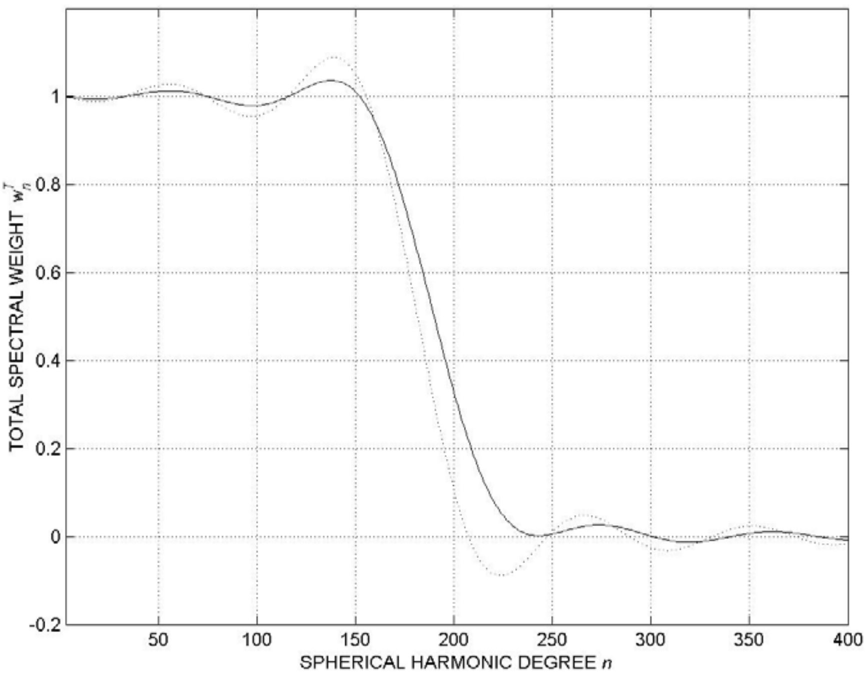

Figure 3. Total spectral weights : Wong-Gore's Ideal filter (*dotted line*) and Butterworth filter (*solid line*)

5. CONCLUSIONS AND RECOMMENDATIONS

A procedure to design the combination solution of a global potential model and terrestrial gravity anomalies based on Molodensky's approach which is adapted to Jawa island characteristics has been prepared. A potential promising candidate of a suitable spectral weighting scheme to weigh both data sets is by means of Butterworth filter. It reduces significantly the propagation of truncation error to the estimated height anomaly, and at the same time it also may also reduce effectively the long-wavelength errors contained in gravity anomalies. On the other hand, the error propagation of the low-degree coefficients of the global geopotential model is not reduced. However, when using a very accurate global geopotential model the error will be relatively small.

By the use of Molodensky's approach, we get the quasigeoid instead of the geoid. Nevertheless, accurate geoid can be derived from accurate quasigeoid when assuming a detailed crust's density information in the island is available.

Following this study, the next step can be several investigations to test the proposed procedure in a small area where dense and accurate gravity anomaly data are available. The investigation should more concentrate on modelling refinement to yield a more optimal and suitable geoid determination procedure. Besides, a comprehensive study on new gravity measurements covering the whole island of Jawa for geoid determination purpose is also strongly recommended.

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